Modeling and Simulation Heat Radiation Transfer Equation during Nanoparticles treatment of Tumor by Laser

Ali Madfa & Fahm Yehya*

Department of physics , Faculty of Education and Science ,Albaydha university , Yemen .

**fahembajash@gmail.com*

Abstract

In this study, we developed an advanced mathematical model to describe the propagation of thermal radiation from a laser within a tumor, without relying on nanomaterial-based treatments. All relevant parameters were incorporated in to the equation, such as absorption, conduction, metabolism, and thermal diffusion rate .We also explored the concept of thermal penetration depth d_{eff} and its correlation with radiation exposure time, integrating it into the solution equation, which allowed us to generate accurate temperature distribution curves within the tumor. Furthermore, we introduced the concept of nanoparticle absorption efficiency within the tumor A_{abs} , where we determined the tumor's overall absorption coefficient $\mu_{a,tot}$. This coefficient was then linked to the presence of nanomaterials in the temperature distribution equation, representing an innovative and novel addition compared to previous research, similarly equation 25 is practically used in laser-based ablation treatment of tumors to determine the radiation dose and exposure time, based on the properties of the nanomaterial used in the treatment ,the type of targeted tissue, and the maximum temperature in side the tumor. We also performed a Monte Carlo simulation of the temperature distribution equation, where the results demonstrated complete alignment with the proposed mathematical model.

***Keywords** : laser ; tumor ; distribution; modeling ; parameter; temperature, Monte Carlo, Simulation ; nanoparticle.*

نمذجة ومحاكاة معادلة انتقال الاشعاع الحراري خلال العلاج النانوي للورم بالليزر

الملخص

في هذه الدراسة، طورنا نموذجا رياضيا متقدما لوصف انتشار الاشعاع الحراري الناتج عن الليزر داخل الورم، دون الاعتماد على العلاجات النانوية. تم تضمين كافة المعاملات ذات الصلة بالمعادلة، مثل الامتصاص، الأيض، ومعدل الانتشار الحراري.

كما استعرضنا مفهوم عمق الانتشار الحراري d_{eff} وارتباطه بزمن التعرض للإشعاع، وأدمجناه في معادلة الحل، مما أتاح لنا رسم منحنيات دقيقه لتوزيع درجه الحرارة داخل الورم.

اضافه الى ذلك، قمنا بتقديم مفهوم فعالية الامتصاص للجسيمات النانوية داخل الورم A_{abs} ، حيث تم تحديد معامل الامتصاص اللي للورم $\mu_{a,tot}$. وقد تم ربط هذا المعامل بوجود المواد النانويه في معادلة توزيع درجة الحرارة مما يمثل اضافه مبتكره وجديده مقارنه بالأبحاث السابقة.

كذلك تم الاستفادة من المعادلة 25 عمليا في العلاج الجراحي للورم عبر الليزر من خلال تحديد كمية الجرعة الإشعاعية وزمن التعرض للإشعاع حسب خواص المادة النانوية المستخدمة في العلاج ونوع النسيج المستهدف وأقصى درجه حرارة داخل الورم.

كما نفذنا محاكاه حاسوبيه باستخدام طريقة مونتكارلو لمعادلة توزيع درجة الحرار ة، حيث أظهرت النتائج توافقا تاما مع النموذج الرياضي المقترح

الكلمات المفتاحية: الليزر، الورم، التوزيع، نمذجة، بارامترات، درجة الحرارة، مونتكارلو، محاكاه، جسيمات نانوية.

Introduction1

Due to the importance of heat radiative equation of the laser and its governance in studying the temperature distribution inside the tumor and its effect on healthy tissues, also the intensity equation of the laser beam follows the uniform Gaussian distribution the thermal diffusion follows the distribution itself, taking into account and the parameters affecting the propagation and the difficulty of the direct mathematical solution to the diffusion equation .

As most studies are computer simulations that are not a mathematical solution rule for that equation that had to be modeled as a mathematical solution from which researchers proceed to be determine the actual spread and determine the places of damage thermal for healthy tissues. In this study a mathematical solution model was developed for the diffusion equation the results of which were identical to the results of previous studies and it can be applied within a certain range or limits.

There are number of methods are a suitable to solve the radiative transfer equation (RTE) , finite volume method (FVM) formulated by Chai et al. Monte Carlo (MC) finite element method (FEM) , discrete ordinates method (DOM) developed by Chandrasekhar for stellar applications was used by Carlson and Lathrop .[1]

2 – Radiative transfer Equation

The radiative transfer equation (RTE) can be written as follows [4]:

$$\vec{\Omega}\nabla I_\lambda(\vec{r},\vec{\Omega}) + \mathcal{B}_\lambda I_\lambda(\vec{r},\vec{\Omega}) = \frac{\sigma_\lambda}{4\pi}\int_{4\pi} I_\lambda(\vec{r},\vec{\Omega}')\,\Phi_\lambda(\vec{\Omega}'\vec{\Omega})\,d\vec{\Omega}' + S(\bar{r},\hat{\Omega}) \quad (1)$$

Where

$S(\bar{r},\hat{\Omega})$: Expected particle source in phase space[2]

Nomenclature

ψ	angular flux in phase space
E	particle energy
Ω	sold angle
r	position vector
S	particle source
I	intensity of laser
λ	wave length
Φ_λ	phase function
μ_a	absorption coefficient of tissue
$\mu_{a,np}$	absorption coefficient of nanoparticle
μ_s	scattering coefficient
d	radius of laser
C	specific heat of tissue
K	thermal conductivity
d_0	tumor depth position
T_0	initial temperature
ρ	density of tissue
ρ_b	density of blood
ω_b	blood perfusion rase
κ	temperature conductivity
Q_r	radiative energy
Q_{met}	Metabolic energy
Q_{laser}	laser energy
τ_l	laser irradiation time
t	time

C_b	specific heat of blood
$б_{abs}$	cross-section absorption of nanoparticles
$б_{sct}$	cross-section scattering of nanoparticles
A_{abs}	Absorption efficiency
A_{sca}	Scattering efficiency
r_n	radius of nanoparticles
C_{np}	Concentration of nanoparticles
N	Number of nanoparticles per unit of volume

3 - Heat Radiative Equation

If we start to create a mathematical model based on thermodynamics, the model has three parts: heat generation ,heat transport , heat effects ,and in order to derive or design this model, we have to take into account several parameters: laser parameters,
thermal tissue parameters .

Heat generation 3 .1

3.1.1 Laser Source [5]

There are three techniques that use thermal surgical treatment to destroy cancer cells
one of them is laser therapy , in addition to radiofrequency and microwaves waves
Laser Ablation technology (LA) involves using lasers to coagulate or destroy tissues. Several techniques have emerged in this field, the most notable being the Laser induced interstitial tumor therapy (LIITT) .in superficial treatment, CO2 lasers used with wavelengths reaching up to 1060 nm ,most Laser Ablation (LA)procedures use Nd: YAG with a wavelength of 1064 nm or semiconductor diode lasers with a(800 – 980 nm) operating in the range of 2 – 40 Watt and use 110/220 V . These lasers do not require pump lamp or water cooling . LA Technology utilizes two types of waves
continuous waves CW and pulsed waves PW .
In the CW,the laser power level ranges 1 to 30 W ,with a treatment duration of a few
minutes up to a maximum of 20 minutes .In contrast , PW emit laser energy in intermittent bursts ,in the form of a series of pulses , where the power level in each pulse is greater compared to that in the continuous wave.
This wavelength is within the thermal radiation regen of the electromagnetic spectrum
which extends between 100 nm and 100 μ m within the spectrum[6]. In general NIR waves are the most widely used part of the electromagnetic spectrum.[5]
When the tissues are exposed to laser beams, heat is generated inside the tissue. In the absence of scattering in the medium along the Z axis , the heat deposited per unit area per unit time in thickness Z is given by the relation: [7]

$$Q_{laser}(r,z,t) = \frac{I(r,z,t)-I(r,z+\Delta z,t)}{\Delta z} \qquad (2)$$

when the value of Δz turns to zero

$$Q_{laser}(r,z,t) = -\frac{\partial I(r,z,t)}{\partial z} \qquad (3)$$

Taking into consideration all the circumstances

$$Q_{laser}(r,z,t) = \mu_a I(r,z,t) \qquad (4)$$

where the value of I was calculated from Beer-Lambert's law as follows :[8]

5) $I(z) = I_0 e^{-r^2/2d^2} . e^{(\mu_s z)} . e^{-(\mu_a+\mu_s)z}$

$$Q_{laser}(r,z,t) = \mu_a I_0 e^{\frac{-r^2}{2d^2}} * e^{(\mu_s z)} * e^{-(\mu_a+\mu_s)z} \qquad (6)$$

Where I is the laser intensity $(W.mm^{-2})$, I_0 is the initial intensity at tissue surface $W.mm^{-2}$, μ_a is the absorption coefficient of the tissue (m^{-1}), $\mu_s$is the scattering coefficient (m^{-1}) z is the depth of the tissue (mm) , and d is the radius of laser beam
(m) where we notice that the heat of the source is function of absorption coefficient and local intensity that is determined from beer-lambert's law.

3.1.2 Nanoparticles

Nanoparticles represent a significant role in bio-diagnostic , imaging and treatment many

diseases affecting human health nanoparticles enable the use of laser light in a targeted and precise manner ,NP-assisted laser inactivation of CD8-specific white blood cells. These nanoparticles can be engineered to be sensitive to specific wave-lengths of light allowing for effective delivery of laser energy directly to cancer cells.

Nanoparticles respond to thermal radiation that reaches them by absorbing a signif-icant portion of it, which depends on the type and shape of the nanomaterial.

a- type and shape of nanomaterial [9,10,11]:

There are many nanomaterials used in radiation therapy for tumors , with gold nanoparticles(GNPs)and silver nanoparticles(Ag-NPs), Iron oxide NPs are used in resonance imaging , Carbon nanotubes a kay role in drugs delivery, Q D NPs have been used to detect the location off malignant cells inside the body .[11]

Gold nanoparticles possess optical properties that enhance the effect of surface plasmonic response (S PR) , they are used in photothermal therapy with laser radiation at a wavelength of 1064 nm .In 2007,Xia and developed gold nanoparticles and used them to eliminate cancer cells. [10]

Gold nanoparticles can infiltrate tissues extensively to reach the tumor area [10].

Experimental results have shown that there is an increase in temperature ranging from 10 to C° 50 C° during c w laser irradiation[12] .

Silver nanoparticles have distinctive technological and chemical applications in the medical field due to their chemical and physical properties. They have a response range to wavelengths between 400 nm and 450 nm [12]

The amount of energy absorbed by nanoparticles and optical properties is given by the following relationship : [13]

$$Q_{abs-np} = c_{abs}\, NI$$

$$\mu_{s,tot} = \mu_{s,tissue} + \mu_{abs,np}$$

[13] $\mu_{a,tot} = \mu_{a,tissue} + \mu_{abs,np}$

Where Q_{abs-np} is absorbed energy of laser in tissue and nanoparticles

I : intensity of laser

μ_{abs-np} : absorption cofficient (m^{-1})

c_{abs} : absorption cross − section (m^2)

: radius of nanoparticles (nm)[14] $r_{eff} = \left(\frac{3}{4\pi}V\right)^{1/3}$

N : number of nanoparticles per unit volume (particle / m^3)

A_{abs}: absorption efficiency

3.1.3 Surface plasmonic response SPR

It is the phenomenon of collective oscillation of free electrons on the surface of a material , occurring when the size of the medium is similar to the wavelength of light at the same frequency . Free electrons oscillate collectively with light , the Mie developed his theory to explain light scattering based on its interaction a size comparable to the wavelength of the incident light unlike Rayleigh's theory .The effectiveness of extinction , scattering , and absorption was calculated using the following relationships : [15],[16]

$$A_{ext} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)Re(a_n + b_n)$$

$$A_{sca} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)(a_n^2 + b_n^2)$$

$$A_{abs} = A_{ext} - A_{sca}$$

$$a_n = \frac{m\psi_n(mx)\psi_n'(x) - \psi_n(x)\psi_n'(mx)}{m\psi_n(mx)\xi_n'(x) - \xi_n(x)\psi_n'(mx)}$$

$$b_n = \frac{\psi_n(mx)\psi_n'(x) - m\psi_n(x)\psi_n'(mx)}{\psi_n(mx)\xi_n'(x) - m\xi_n(x)\psi_n'(mx)}$$

Here x is the size parameter given by $x = 2\pi n_m R/\lambda$, and m is ratio of refractive index of the sphere n to that of the surrounding medium n_m , $m = \varepsilon/\varepsilon_m$, ε is dielectric function, ψ_n, ξ_n are the Bessel functions. [16]

3.2 Heat transport [7]

In the actual (real) interaction between laser and tissue , all the variables of thermal transfer must be studied.

Heat convection : is heat transport in tissue due to blood flow ,is given by: [17]

$$Q_b = \rho_b \omega_b C_b (T_b - T) \qquad (7)$$

Where

: heat caused by blood flow (W/ m^3) Q_b

ρ_b : density of blood (kg / m^3)

ω_b: blood perfusion rate of the tissue (s^{-1})

T_b : temperature of the blood (C°)

Heat radiation :is defined by Stefan-Boltzmann law which the radiated power is

related to the fourth power of temperature , can be expressed by: [6] .

$Q_r = \sigma T^4$ (8)

σ : Stefan -Boltzmann constant (5.67 x10^{-8} $Wm^{-2}K^{\circ -4}$)

Heat metabolic : it is the metabolic heat generated in the tissue and varies according to the type of tissue .[18]

4- Heat effect [7]

When study in thermal distribution within biological tissue initial value and boundary conditions should be studied . It was assumed that the human body temperature is 37C° and when it rises to five degrees , there are nothing of concern that can be measured. When the tissue temperature reaches 50C°,there is a clear effect that can be observed and measured .There is a decrease in the effectiveness of the enzyme and results in a decrease in the energy transported inside the cell, which causes the cells to freeze .within 60C°, protein denaturation and collagen occurs which leads to tissue coagulation and local cell death. At 80C° the permeability of the membrane increases destroying the preservation of the balance in the chemical concentration in any way. At100 C° the water molecules that make up most of the tissue begin to evaporate, as a result of an increase in volume during this transport phase with a dynamic rupture and thermal decomposition of tissues . If all the water molecules evaporate and the exposure to laser radiation is continuous , there will be an advanced increase in temperatures,and the charring of the tissues will take its position and cause blackening in the adjacent tissues. for molecules ,cells to remain effective at the temperature level Arrhenius put an equation[19] :

$$\Omega(t) = \ln \frac{C(0)}{C_t} = A \int_0^t \exp(-\frac{E}{\mathcal{R}T(t)})dt \quad (9)$$

Where

Ω : Arrhenius thermal damage

C_0 : initial concentration of molecules and cells

A : Arrhenius constant.

R : the universal gas constant (J / mol .K°)

E : activation energy (J /mol)

5- Temperature distribution in tissue

The change in heat content dQ motivates an alteration with the temperature as follows

$dQ = mc\, dT$ (10)

where m is the tissue mass , and c is the specific heat capacity ($KJ\, kg^{-1}K^{\circ -1}$) by for most tissues is given by (c = 4.35 $KJ\, kg^{-1}K^{\circ -1}$) .

The heat flow is proportional to the gradient in temperature and according to the diffusion equation :[7]

$J_Q = -k \nabla T$ (11)

Where k is heat conductivity ($Wm^{-1}K^{\circ -1}$) .

the derivative of heat content per unit volume of tissue with respect to time is equal to the amount of divergence of heat flow within the tissue.

$\text{div}\, J_Q = -\dot{q}$ (12)

equation (11) can be written as follows

$$\rho C \dot{T} = \frac{\dot{Q}}{V} = \dot{q} = -\text{div}\, J_Q \quad (13)$$

Adding the source heat Q_{laser},the heat flowing through the blood Q_b,and the radiation heat to the equation (13)

$$\rho C \frac{\partial T}{\partial t} = K\nabla^2 T + \rho_b C_b \omega_b (T_b - T) + Q_r + Q_{met} + Q_{laser} \quad (14)[8]$$

where ρ is density of tissue (Kg/m^3) , C is heat capacity ($J.Kg^{-1}k^{\circ -1}$), ω is blood perfusion rate (s^{-1}) , Q_{met} is metabolic heat source ,T is Temperature (C^0) , K is thermal conductivity of tissue ($W.m^{-1}, k^{\circ -1}$) , Q is heat generation ($W.m^{-3}$) [8]

where

∇^2 Laplace operator

$\nabla^2 = \nabla^2 x + \nabla^2 y + \nabla^2 z$

6- Modeling solution of heat radiation equation

6.1.1 Without nanoparticles

A cording to the laws of thermodynamics ,the change in heat content occurs as a liner change

in temperature in any close thermodynamic system ,i.e.

$$dQ = mc\ dT$$

$$dT = \frac{1}{mc}\ dQ$$

$$dT = \frac{1}{\rho cV}\ dQ$$

It is know that the temperature of the human body 37C°, and therefore the change stats from higher values , and to find the temperature inside the tissue at any moment in time for thermal radiation when a laser source is shed , through the integration process of the equation , where the radiation power of laser is a function of the exposure time , of different derivatives laws can be applied with the following relationship :

$$df = \frac{\partial f}{\partial t}\ dt \qquad [20]$$

$$dQ = \frac{\partial Q}{\partial t}\ dt$$

$$dQ = \dot{Q}\ dt$$

$$dT = \frac{1}{\rho C}\frac{\dot{Q}}{V}\ dt$$

At $\frac{\dot{Q}}{V} = Q_{laser}$

Where Q_{laser} : Laser energy (W / m^3)

$$\int_{T_0}^{T} dT = \frac{1}{\rho C}\int_0^{\tau_l} Q_{laser}\ dt$$

$$T - T_0 = \frac{1}{\rho C}\ Q_{laser}\ \tau_l$$

$$\Delta T = T - T_0 = \frac{1}{\rho C}\ Q_{laser}\ \tau_l \quad (15)$$

Where

$$Q_{laser} = \mu_a I$$

at $I = I_0$, I_0 *intial intensity of laser peam*

$$\Delta T = T - T_0 = \left(\frac{\mu_a I_0}{\rho C}\ \tau_l\right) = T_{max} \quad (16)$$

The equation (16) represents the maximum change value temperature within the tissue, I.e. The maximum value of the temperature within that tissue at the time of laser.

Since the temperature changes in side the tissue according to the parameters affecting the tissues ,where the intensity of radiation follows a Gaussian distribution , it was necessary to find a function that contains all these parameters as follows:

$$\Delta T = T_{max}\ f(x,y,z,t,\mu_a,\omega_b,Q_{met},\rho,\mathrm{C},\dots.)$$

$$f(x,y,z,\dots) = p(x,y,z,r,\dots) = \frac{\Delta T}{T_{max}}$$

That is , this function is the density of the probability function pdf.

$$\Delta T = T_{max}\ p(x,y,z,\dots)$$

Where

$$p(x,y,z,..) = \prod_{i=1}^{n} exp\ (-\mu_i)$$

$$p\ (x,y,z,..) \propto \left(exp - \left(\frac{r^2+z^2}{8{d_0}^2}\right) + \exp - \left(\frac{r^2+{d_0}^2}{4\kappa t}\right)\right)$$

$$p\ (x,y,z,..) \propto \cos(\omega_b t)$$

$$p\ (x,y,z,..) \propto \exp - \left(\frac{Q_{met}d^2}{\mathrm{K}T_b}\right)$$

$$p\ (x,y,z,..) \propto \cos(\omega_b t)\ \left(exp - \left(\frac{r^2+z^2}{8{d_0}^2}\right) + \exp - \left(\frac{r^2+{d_0}^2}{4\kappa t}\right)\right) \exp - \left(\frac{Q_{met}d^2}{\mathrm{K}T_b}\right)$$

$$p\ (x,y,z,..) = g\ \cos(\omega_b t)\ \left(exp - \left(\frac{r^2+z^2}{8{d_0}^2}\right) + \exp - \left(\frac{r^2+{d_0}^2}{4\kappa t}\right)\right) \exp - \left(\frac{Q_{met}d^2}{\mathrm{K}T_b}\right)$$

Where g is a mathematical constant given by

$$g = \begin{cases} 1 & r,z \neq 0 \\ \left(\frac{\tau_l}{a*t+\sqrt[4]{t}}\right) & r = z = 0 \end{cases}$$

a=1 when $t \geq 10\ sec$

$1 > a > 0$ when $t \leq 10\ sec$

$$p\ (x,y,z,..) = g\ \cos(\omega_b t)\ \left(exp - \left(\frac{r^2+z^2}{8{d_0}^2}\right) + \exp - \left(\frac{r^2+{d_0}^2}{4\kappa t}\right)\right) \exp - \left(\frac{Q_{met}\,d^2}{\mathrm{K}T_b}\right)$$

$$\Delta T = g\ T_{max}\ \cos(\omega_b t)\ \left(exp - \left(\frac{r^2+z^2}{8{d_0}^2}\right) + \exp - \left(\frac{r^2+{d_0}^2}{4\kappa t}\right)\right)\ \exp - \left(\frac{Q_{met}d^2}{\mathrm{K}T_b}\right)$$

$$T(r,t) = T_0 + T_{max}\ g\ cos(\omega_b t)\ \left(exp - \left(\frac{r^2+z^2}{8{d_0}^2}\right) + exp - \left(\frac{r^2+{d_0}^2}{4\kappa t}\right)\right) exp - \left(\frac{Q_{met}d^2}{\mathrm{K}T_b}\right) \quad (17)$$

Thermal diffusion depth : it is know that traditional method of treating tumors with laser causes the spread of temperature to the surrounding tissue at temperatures close

to those of the tumor itself. This aspect is often neglected when studying the thermal distribution within tissues . It was necessary to include this influential factor in theoretical calculations , so we introduced what is known as the thermal diffusion depth ,which is given by the following relation :[4]

$$d_{th} = \sqrt{4\kappa t} \qquad (18-a)$$

and at a large time t ≥ 5 minutes can be written :

$$d_{th} = \sqrt{\kappa t} \qquad (18-b)$$

This depth increases with exposure time of radiation , which is why pulsed laser sources have significantly addressed this part of the effect. Since this thermal depth is adjacent to the tumor , we will consider that the tumor thickness (the thickness of the active region) is given by :

$$d_{eff} = d_0 + d_{th}$$

by substituting into equation 30 ,the form of the equation becomes as follows:

$$T(r,t) = T_0 + T_{max}\cos(\omega_b t)\left(\exp - \left(\frac{r^2+z^2}{8{d_0}^2}\right) + \exp - \left(\frac{r^2+{d_{eff}}^2}{d_{th}^2}\right)\right)\exp - \left(\frac{Q_{met}d^2}{KT_b}\right) \qquad (19)$$

by applying the initial values and boundary conditions

r = z = 0

$$T(0,t) = T_0 + \left(\frac{\tau_l}{t+\sqrt[4]{t}}\right) T_{max}\cos(\omega_b t)\left(1 + \exp - \left(\frac{{d_{eff}}^2}{d_{th}^2}\right)\right)\exp - \left(\frac{Q_{met}d^2}{KT_b}\right)$$

The general form of the temperature distribution equation over time given by

$$T(t) = T_0 + \left(\frac{\tau_l}{t+\sqrt[4]{t}}\right) T_{max}\cos(\omega_b t)\left(\frac{1}{+\exp - \left(\frac{{d_{eff}}^2}{d_{th}^2}\right)}\right)\exp - \left(\frac{Q_{met}d^2}{KT_b}\right) \qquad (20)$$

6.1.2 Within nanoparticles

According to the phenomenon of surface plasmon resonance SPR and how electrons respond collectively to light , a specific interaction occurs with the light , causing it to scatter in a particular manner , which enhances both absorption and scattering based on previous models that used on previous models that used mathematical formulas to find the total absorption coefficient in the presence of a nanoparticle ,which relationships according to the studies of: [17]

$$\mu_{a,np} = A_{abs}\,\pi\, r_{np}^2\, N_{na}$$

$$\mu_{s,np} = A_{sca}\,\pi\, r_{np}^2\, N_{na}$$

$$\mu_{a,tot} = (1-\eta)\mu_{a,tiss} + \eta\mu_{a,np}$$

$$\mu_{s,tot} = (1-\eta)\mu_{s,tiss} + \eta\mu_{s,np}$$

Where η is the volume fraction , which is depended on the concentration N_{na} and diameter of the nanoparticles[17] :

$$\eta = \frac{\pi}{6} r_{np}^2\, N_{na}$$

However , in mathematical and practical calculations of the absorption coefficients have very high values , reaching hundreds of times compared to the absorption coefficient of tissue or tumor , which may deviate from the practical temperature values generated . therefore , a more objective mathematical model was developed to better align with mathematical equations are as follows :

$$\boldsymbol{\mu_{a,tot} = \mu_{a,np} + \mu_{a,tissue}}$$

$$\boldsymbol{\mu_{a,tot} = \frac{\mu_{a,np}}{\mu_{a,tissue}}\ \mu_{a,tissue} + \mu_{a,tissue}}$$

Considering that

$$\boldsymbol{A_{abs} = \frac{\mu_{a,np}}{\mu_{a,tissue}}}$$

Therefore

$$\boldsymbol{\mu_{a,tot} = A_{abs}\ \mu_{a,tissue} + \mu_{a,tissue}}$$

$$\boldsymbol{\mu_{a,tot} = (A_{abs} + 1)\,\mu_{a,tissue}}$$

Thus ,the relationship used to determine the total absorption coefficient is given by the equation :

$$\boldsymbol{\mu_{a,tot} = (A_{abs} + 1)\,\mu_{a,tissue}} \qquad (21)$$

The ratio represent the absorption coefficient of nanoparticles within the tissue to the absorption coefficient of the tissue itself .If we calculate the absorption coefficient of nanoparticles outside the tissues, it can reach hundreds of times that of tissue .Theis means that the absorption coefficient of these nanoparticles inside the tissues will be much lower that their coefficient before being injected into the tissue . This is dependent on several factors, such as the concentration of the nanoparticles within tissue,the shape and size of the nanoparticles ,and the characteristics of tissue in terms

of density , thickness, blood flow rate ,and the amount of blood contained within the tissue .Thus ,the absorption efficiency of nanoparticles within the tissue will directly affect the efficiency of absorption of the particles present inside the tissue. From this, we will refer to this ratio as the absorption efficiency of nanoparticle within tissue, which we will consider to be approximately equal to the absorption efficiency of nanoparticles that can be determined through the mathematical relationship in equation 23 .Additionally, we can add the factor $\boldsymbol{g}$, which represents the anisotropy of the tissue, to equation (23) we can also mathematically consider $\boldsymbol{g}$ to be equal to one

given that the absorption efficiency present in the equation represents its maximum value obtained from experimental and practical results , and since this absorption efficiency changes with variations in the transverse absorption value -which depends on both the concentration c_{np} and the effective radius r_{eff} of the mathematically as follows [7] :

$$\mathrm{Q_{abs}} = \frac{1}{ln10}\, Nb\, C_{abs} \qquad (22-a)$$

$$C_{abc} = \mathbf{A_{abs}}\, \boldsymbol{\pi r_{eff}^2} \qquad (22-b)$$ [4]

$$\mathrm{Q_{abs}} = \frac{1}{ln10}\, Nb \mathrm{A_{abs}}\, \pi r_{eff}^2$$

$$\mathbf{A_{abs}} = \boldsymbol{g}\, \frac{\mathbf{2.3\ Q_{abs}}}{\boldsymbol{\pi r_{eff}^2\, Nb}} \qquad (23)$$

When one of the parameters, either concentration or effective radius, is present, we can use the following two relationships:

$$\boldsymbol{A_{abs(r)} = A_{abs,max}\, exp - \left(1 - \frac{r_0}{r}\right)^2} \qquad (24-a)$$

Where

r_0 : *radius of nanoparticles at maximum value of* $A_{abs,max}$

$$\boldsymbol{A_{abs(r)} = A_{abs,max}\, exp - \left(1 - \frac{c}{c_0}\right)^2} \qquad (24-b)$$

Where

c_0 : concentration of nanoparticles at maximum value of $A_{abs,max}$

C : Different concentration values of the nanoparticles

By returning to equation 19 for temperature distribution and substituting into it, it becomes as follows :

$$T(r, t) = T_0 + \frac{I_0}{\rho C}(A_{abs} + 1)\mu_{a,t}\tau_l \cos(\omega_b t)\left(exp - \left(\frac{r^2+z^2}{8{d_0}^2}\right) + \exp - \left(\frac{r^2+{d_{eff}}^2}{d_{th}^2}\right)\right) \exp - \left(\frac{Q_{met}d^2}{KT_b}\right)$$

According

$$T(r, t) = T_0 + \frac{I_0\mu_{a,t}\tau_l}{\rho C}(A_{abs} + 1) \cos(\omega_b t)\left(exp - \left(\frac{r^2+z^2}{8{d_0}^2}\right) + \exp - \left(\frac{r^2+{d_{eff}}^2}{d_{th}^2}\right)\right) \exp - \left(\frac{Q_{met}d^2}{KT_b}\right)$$

From equation (19)

$$T(r, t) = T_0 + T_{max}(A_{abs} + 1) \cos(\omega_b t)\left(exp - \left(\frac{r^2+z^2}{8{d_0}^2}\right) + \exp - \left(\frac{r^2+{d_{ef}}^2}{d_{th}^2}\right)\right) \exp - \left(\frac{Q_{met}d^2}{KT_b}\right) \qquad (25)$$

and when applying the initial and boundary conditions , the equation becomes:

$$T(t) = T_0 + T_{max}\, g(A_{abs} + 1) \cos(\omega_b t)\left(1 + \exp - \left(\frac{(d_{eff})^2}{d_{th}^2}\right)\right) \exp - \left(\frac{Q_{met}d^2}{KT_b}\right)$$

Therefore

$$T(t) = T_0 + T_{max}\left(\frac{\tau_l}{t+\sqrt[4]{t}}\right)(A_{abs} + 1) \cos(\omega_b t)\left(\frac{1}{+\exp - \left(\frac{(d_{eff})^2}{d_{th}^2}\right)}\right) \exp - \left(\frac{Q_{met}d^2}{KT_b}\right) \qquad (26)$$

7. Result and Discussion

7-1-1 Without nanoparticles

In the study conducted by Wongchadakul et al ., and based on the parameters we observe that the thermal distribution within the tissue is plotted as a function of (r) entered as shown in the table 1. Where the Monte Carlo simulation method was used under different absorption coefficients and varying radiative intensities.[8]

parameters	Tumor
Tissue density ,ρ (Kg.m^{-3})	1030
Specific heat of tissue ,C(J /Kg .k)	3852

Thermal conductivity .K(W/ m .k)	0.558
Blood perfusion ω_b (s^{-1})	0.00063
Metabolic heat generation , Q_{met} (W.m^{-3})	3680
Absorption coefficient ,μ_{abs} (m^{-1}),532 nm	4 , 10 , 15
The radius of laser beam ,d (mm)	2
Tumor depth position	3 , 8
Blood temperature ,T_b (C^o)	37
Initial temperature , T_0 (C^o)	37
Laser irradiation intensity , I (W.mm^2)	1 , 2
κ temperature conductivity ($\frac{m^2}{sec}$)	1.4e-4

Table. 1 . biology and optical properties of Tissue at Wongchadakul et al study[8].

Figures (1-a) illustrates that the temperature reaches 100 ºC at the tumor center when the tumor absorption coefficient is 15 m^{-1} ,and the laser intensity is 1 w.m^{-2} where temperature reaches 80 ºC when the absorption coefficient is 10 m^{-1}. And reaches 50 ºC when absorption coefficient is 4 m^{-1}. Figures (1-b) illustrates that the temperature reaches 130 ºC when the tumor absorption coefficient is 15m^{-1} ,and laser intensity is 2 w.m^{-2} , and reaches 100 ºC when the tumor absorption coefficient is 10m^{-1} ,and reaches 70 ºC when the tumor absorption coefficient is 4m^{-1} .

Figures (2-a) illustrates that the temperature distribution within tissue at depth 8 mm where temperature reaches 85 ºC when the tumor absorption coefficient is 15 m^{-1} and reaches 75 ºC when the tumor absorption coefficient is 10m^{-1} , reaches 50 ºC when the tumor absorption coefficient is 4 m^{-1} where laser intensity is 1 W. m^{-2}.

a b

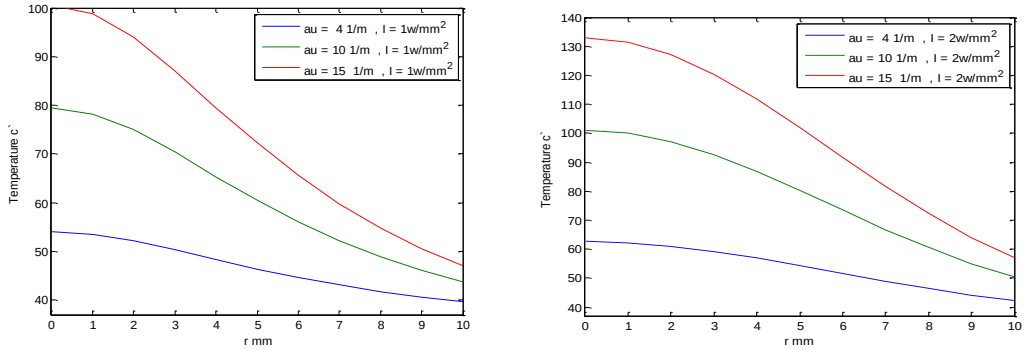


Fig .1 Temperature distribution in tissue at various absorption coefficient at current study.

Also ,we observe from Figure 2-b ,where illustrates that the temperature reaches 130 ºC at the depth 8 mm when the tumor absorption coefficient is 15 m^{-1} , and the laser intensity is 2 w. m^{-2} ,where temperature reaches 95 ºC when the absorption coefficient is 10 m^{-1}. And reaches 65 ºC when absorption coefficient is 4 m^{-1}. Where we notice the 1-b illustrates that the temperature reaches 130 ºC at the center of the tumor , when the tumor absorption coefficient is 15m^{-1} , and laser intensity is 2 w.m^{-2} ,reaching 130 ºC in the healthy tissues adjacent to the tumor, which is much higher than the critical temperature (the lowest temperature at which physiological changes in the tissue begin to occur) , indicating that the damage factor is much greater than one $\Omega(t) \gg 1$, and reaches 100 ºC when the tumor absorption coefficient is 10m^{-1}, reaching 85 C^o in the healthy tissues adjacent to the tumor , which is much higher than the critical temperature also .and reaches 70 ºC when the tumor absorption coefficient is 4 m^{-1},reaching 55 ºC in the

healthy tissues adjacent to the tumor which still represents a threat to he healthy tissues , as we also observe in Figure 5 .

a b

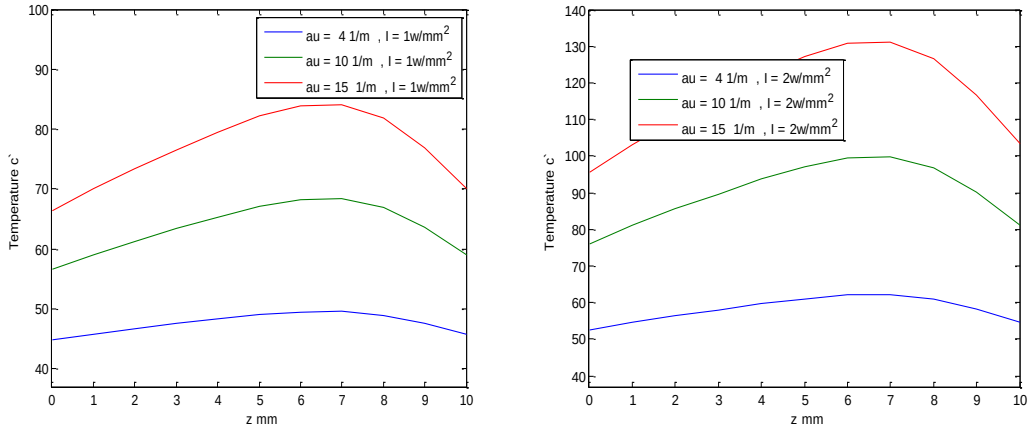
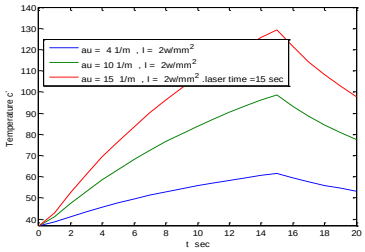


Fig . 2 temperature distribution at a depth within the tissue with different absorption coefficient at present study.

applying equation 20 with the parameters provided in table 6 yields the curve and the boundary conditions provides the temperature curve as shown in Figure 3 .



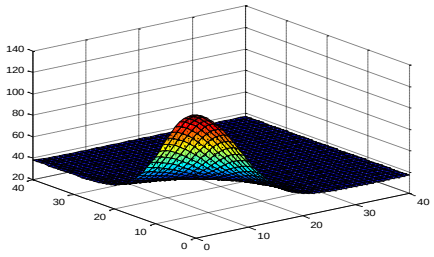
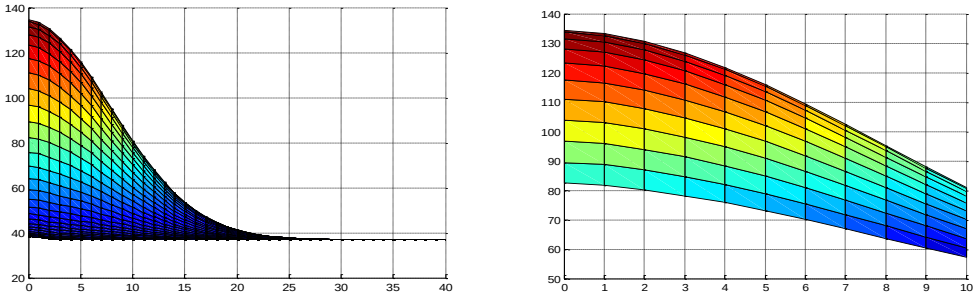


Fig 3 Temperature distribution in tissue at the time . **Fig 4** Three – dimensional distribution of temperature at 1 w . m^{-2} .by present study.

Figure .3 shows the temperature distribution over time , with the maximum value reached at the highest exposure time to the laser (t=15 sec) ,after which it decreases over time once the laser source is turned off. And determines how the temperature is distributed as a function of the laser exposure time.

The figure 4 represent a 3D model of the temperature distribution inside the tumor, with the maximum temperature occurring at the center of the tumor at 1 wm^{-2}, which corresponds to the coordinates (0,0)

The curves in figures 1 , 2 , and 3 , which represent the temperature distribution on the vertical axis in terms of tumor diameter, depth, and irradiation time on the horizontal axes ,were obtained from equations 19 and 20. These curves are in exact agreement with those presented in the study by Wongchadakul et al., and one can refer to the content of that study to observe the matching temperature distribution curves with those we obtained.



By present study **Fig .5** Temperature distribution at center , and at 10 mm of tumor
Figure 5 displays the temperature at the tumor center with an intensity of 2 w.m^{-2} in the right part of the figure at 10 mm , while the left part shows the temperature distribution at the center of tumor .
When applying equation 9 and substituting with equation 20, and plotting the damage a function of time at tissue temperature for laser intensities of 1 , 2m^{-2},we observe the following : at an intensity of 1 w. m^{-2} and absorption coefficient of 4m^{-1} ,the damage rate is much less than one , with the maximum temperature at the tumor center being around 55 ºC. Meanwhile ,it decreases in the healthy tissues to approach 40 ºC which is evident in the left part of Figure 6-a and 6-b .conversely ,the right part of Figure shows that the damage rate is greater than one , reaching up to 50, which is significantly higher than one

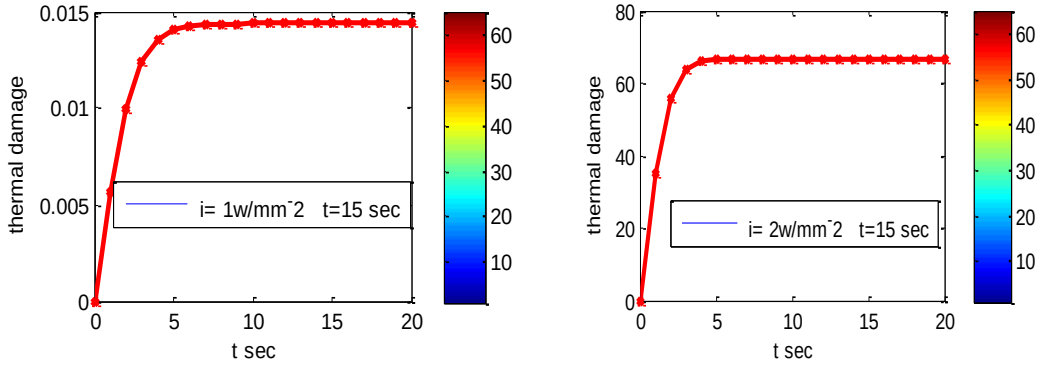


Fig .6-a Thermal damage to tissues on the vertical axis with time on the horizontal axis at 1 ,and 2 $w.m^{-2}$

When calculating the thermal dose within the tumor and the surrounding tissue in terms of tissue temperature, we observe the thermal energy as a result of the radiation
This can cause damage to the tissues surrounding the tumor. Figure 6-b illustrates the amount of thermal dose within the tissue with change in temperature.

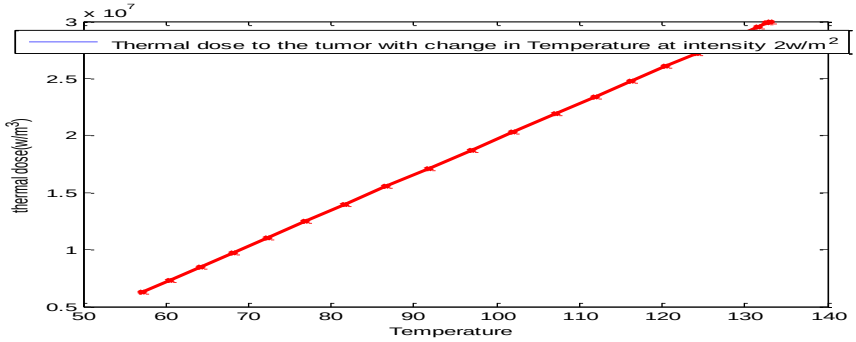


Fig 6-b Thermal dose at intensity 2 w / mm ^2.

7-1-2 Simulation by Monte Carlo :

7-1-2-a : The first method

Let's start by defining the path length of a photon as it moves freely before it begins to undergo any collisions. We need to determine the probability that photon moves freely a distance s followed by a collision in d s , therefore , the probability of the combined effect is given:[2]

P(s)d s = $\mu_t e^{-\mu_t s}\, ds$

Where μ_t is Total coefficient

s = $\frac{-1}{\mu_t}$ ln (1- ξ)

where ξ is Random number

we need to determine the scattering angle θ polar angle and Φ azimuthal angle

where

cos θ = 2 ξ – 1

Φ = 2π ξ

To setting up the geometric graph and its boundary conditions, it is essential determine the position of each particle within the system using the coordinates x, y , z

$x = x_0 + s\,u$

$y = y_0 + s\,v$

$z = z_0 + sw$

Where u ,v ,w are direction cosine of direction along x , y , z axis:[2]

$u = \sin\theta\cos\Phi$

$v = \sin\theta\sin\Phi$

$w = \cos\theta$

$r = \sqrt{x^2 + y^2 + z^2}$

The weight of the biased particle is given by

W = $W_0 \frac{\mu_t}{\mu_a} \exp(-\mu_a r)$ [2]

We will use the Monte Carlo method to study the temperature distribution within tissue,utilizing the parameters used in Wongchadakul et al.,previous study as detailed in table 1 . we observe from the left part of figure 7-a, that we obtained a maximum temperature that matches exactly with the mathematical modeling. similarly, the right Part of the figure ,which shows the temperature distribution over time, indicates that The temperature increases to reach a maximum value of 101 ºC at an exposure time of 15 seconds ,which is a difference of 2 ºC . Were the left part of the figure, which shows the temperature distribution over radius.

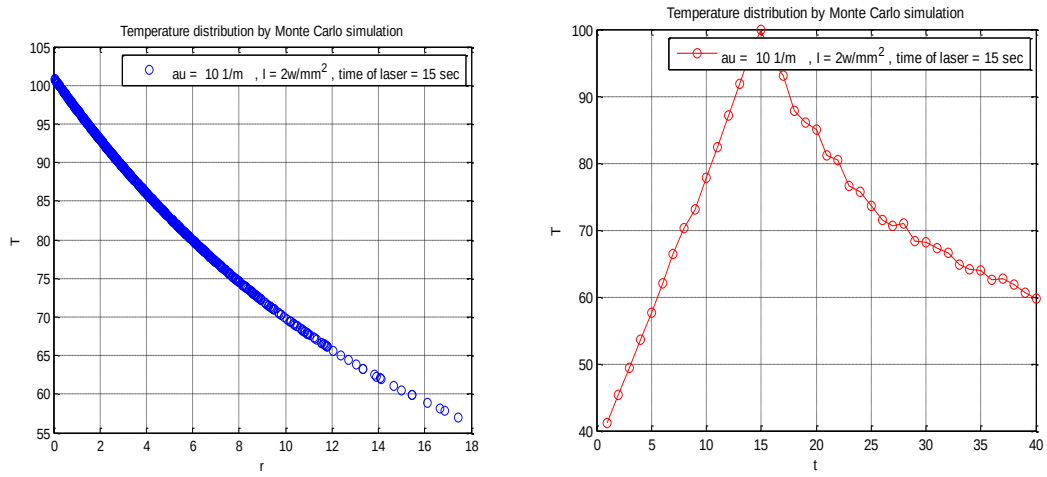


Fig 7-a Temperature distribution in Tumor by Monte Carlo at present study.

Figure 7-b illustrates the Monte Carlo simulation of thermal damage within the tissue Where the damage rate value is greater than one , indicating the presence of thermal damage to the tissues adjacent the tumor.

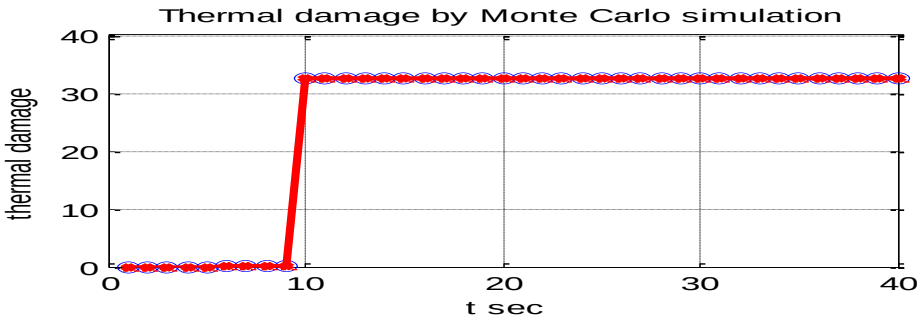


Fig 7-b Thermal damage to the Tissue on the vertical axis versus irradiation time on the horizontal axis at present study .

7-1-2-b : The second methods

In this method, we used the Probability Density Function (PDF) and the Cumulative Distribution Function (CDF) .Using three different values for the number of random numbers(N=100, N=1000 , N=100000), we obtained the following curve.

We can expression by PDF function:

$$p\left(\frac{\Delta T}{T_{max}}\right) dT = \mu\, e^{-(\frac{\mu \Delta T}{T_{max}})}\, dT$$

We can find a variable random by CDF

$$F(\frac{\Delta T}{T_{max}}) = \int_0^T \mu e^{-(\frac{\mu \Delta T}{T_{max}})}\, dT$$

$$\Delta T = T - T_0$$

But $T_0 = 37^0$C

Therefore

$$F(\frac{T-T_0}{T_{max}}) = \eta = \int_{T-T_0=0}^{T} \mu\, e^{-\mu(\frac{T-T_0}{T_{max}})}\, dT$$

Were η : random number

$$\eta = -T_{max}\ [e^{-\mu(\frac{T-T_0}{T_{max}})} - 1]$$

$$\frac{\eta}{T_{max}} = -\ [e^{-\mu(\frac{T-T_0}{T_{max}})} - 1]$$

$$\left[1 - \frac{\eta}{T_{max}}\right] = -\ e^{-\mu(\frac{T-T_0}{T_{max}})}$$

$$-\mu\left(\frac{T-T_0}{T_{max}}\right) = \ln\left[1 - \frac{\eta}{T_{max}}\right]$$

$$\boldsymbol{T - T_0 = -a\frac{T_{max}}{\mu}\ \ln\left[\frac{\eta}{T_{max}}\right]} \quad (27)$$

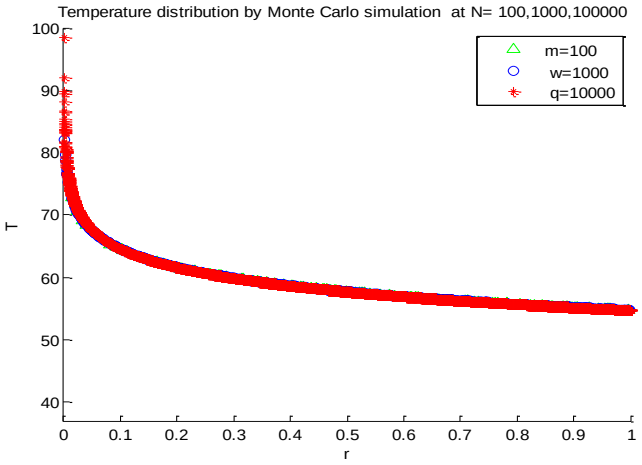


Fig 9 Temperature distribution in Tumor by Monte Carlo at present study

Were (**a**> 1) represents the calibration factor for equation 27 when the absorption coefficient is high .

By applying equation 27 with a radiation intensity of 2 wm^{-2} ,an absorption coefficient of 15 m^{-1} , and an irradiation time of 10 sec , as in the study by Patcharaporn et al., we obtain figure 8 , which represent the temperature distribution within the tumor.

2. within Nanoparticles

In Yildiz et al .(2013) study , we conducted a overview of nanoparticle assisted laser therapy ,where he examined the optical properties of nanoparticles .He calculated the absorption cross-section and the absorption efficiency of NPs, formulated an equation for the absorption coefficient of NPs , and also for the total absorption coefficient when nanoparticles are injected into tissues. He studied GNP and nano polymers and analyzed how these coefficient vary with changes in particle shape and concentration

The laser wavelength was tuned to 808 nm for these experiments and the output power was varied from 0.32 to 1.2 w.[16]

Material	**Dimension(nm)**	**λ_{max} (nm)**	**A_{abs}**	**Laser**	**ΔT (Yildiz et al)^{0}C**	**Δ T (present study)^{0}C**
Au hollow	[40, 50]	980	1.6	4 min ,0.5 w/m^2	16.5	14.857
Si-Au	[120,110]	1120	4.2	6 min ,0.4 w/m^2	37.5	35.65
Nanorod	[14, 15]	797	13.5	3min 0.32w/m^2	32.1	37.028
Nano hexapod		880	13.5	10 min,1.2w/m^2	23.1	--------
Polypyrrole	D=[58]	950	0.75-1	5 min, 1 w/m^2	25	25
PEDOT : PSS	D= [130]	850	0.5 -1	5 min, 0.5 w/m^2	21	14.28

Table .2 presents the results obtained from his study

Upon applying the equations we developed we obtained temperature values within the tumor that closely matched those reported by Yildiz et al ., in their study. Table 2 presents a comparison between the values from the previous study and those obtained in our work. The full content of the prior study may be consulted to compare their results with the findings presented here.

In the study conducted by Xiao et al .(2011), on the numerical investigation of thermotherapy for tumor treatment ,gold nanoparticles of the shell type with diameters of [40,70] nm , and an absorption cross-section of $7.2 * 10^{-14}\, m^2$ were used. concentrations ranging from 0 to $4 * 10^{9}\, ml^{-1}$ were applied., along with an NIR laser with a wavelength of 850 nm .Notably ,varying radiation intensities were used when applied to the three tissues : the prostate , brain, and breast, as well as at different concentrations . [21]

Tissue type	**Specific heat(J/kg K)**	**Density Kg/m^3**	**Absorption coefficient cm^{-1}**	**Wave length(nm)**
Human prostate	3750	1050	0.04	850
Human brain	3600	1035	0.026	633
Human breast	3920	990	0.02	633

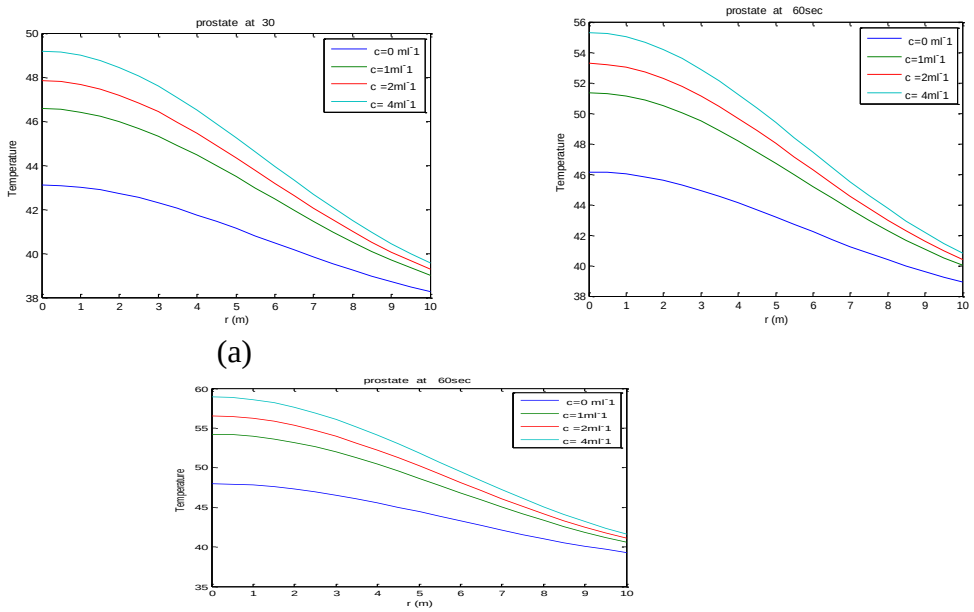
Table .3 biology and optical properties of Tissue. (Xiao et al) [21]

The results obtained from applying equations 24, 25 and 26 are illustrated on the Figure 9 showing the temperature distribution within the prostate at different

Times and concentrations. Additionally ,the absorption efficiency value obtained from equation 23 was equal to 1.2 ,using parameters listed in Table 3 , 4 . the value of the optical path length of the radiation is 20 nm ,and absorbance is 1.6 ,according to the absorbance curve of gold nanoparticles and anisotropy (g = 0.92).

parameters	Tumor
Metabolic heat generation , Q_{met} (W.m^{-3})	3680
Specific heat of tissue ,C(J /Kg .k)	3852
Thermal conductivity .K(W/ m .k)	0.558
Blood perfusion ω_b (s^{-1})	0.00063

Table 4 biology and optical properties of Tissue[8]

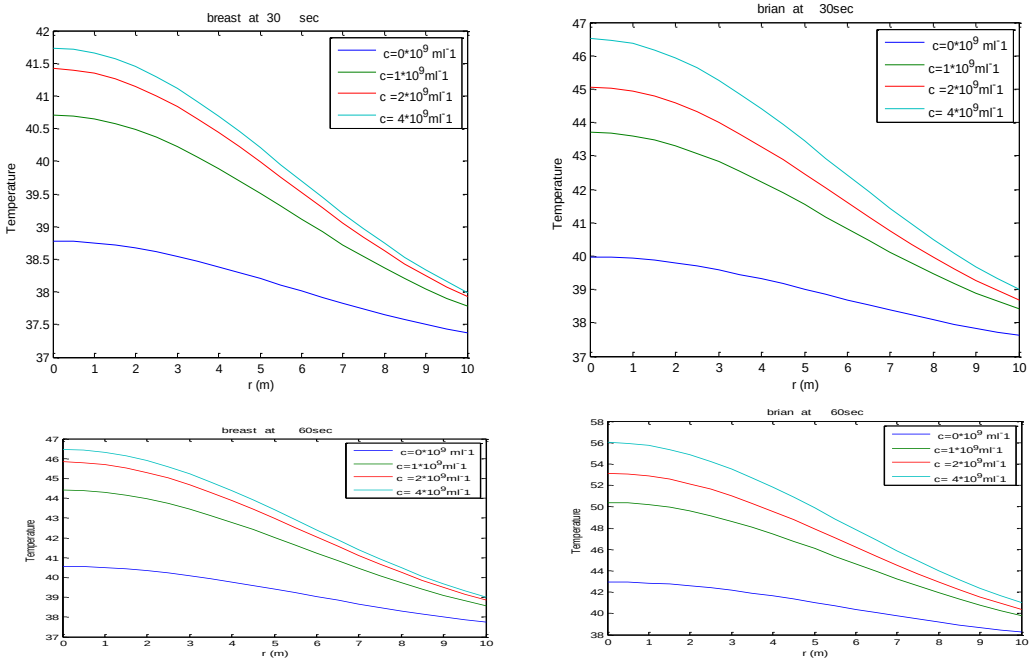


(a) (b)

(c)

Fig .9 Temperature distribution in prostate Tissue within nanoparticles.

The right part of Figure 10 illustrates the distribution of temperature for the breast at 30 sec, 60 sec ,90 sec . The intensity radiation of laser oscillations are swing from 1.5 $W cm^{-2}$ to 2.8 $W\ cm^{-2}$ .While illustrates the left part of Figure 10 the distribution of temperature for the breast at 30 sec, 60 sec ,90 sec . The intensity radiation of laser oscillations are swing from 1.5 $W cm^{-2}$ to 2.8 $W\ cm^{-2}$ where $g = 0.9\ldots$].It can refer to the study of Xiao and observed the shape of the distribution of temperature for tissue. With the comparison of the forms of the distribution of the temperature for previous study and the present study , we note that we have greatly matched the results of the previous study.



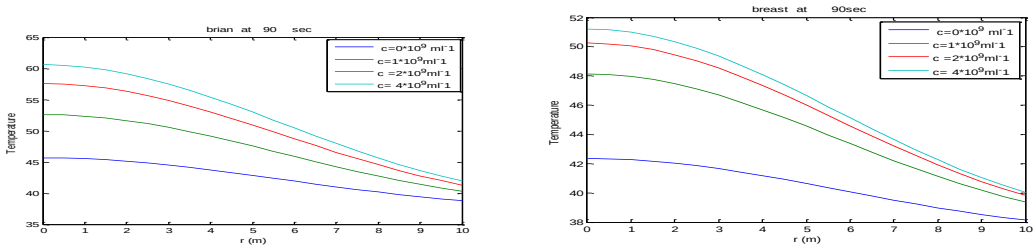


Fig .10 Temperature distribution in brain and breast tissues at present study

. **The curves** in figures 9 , and 10 , which represent the temperature distribution on the vertical axis in terms of tumor diameter on the horizontal axes , were obtained from equations 23 and 24. These curves are in exact agreement with those presented in the study by Xiao et al., and one can refer to the content of that study to observe the matching temperature distribution curves with those we obtained .

In the study conducted by Kim et at (2019) ,in his numerical study on the theory of photothermal effects using a continuous single - mode laser with Gold nanoparticles of type nanorods ,he employed the parameters outlined in Table 4,5 .The results he obtained the maximum temperature within the tissue or tumor reached 72.5 ºC[22] .

Parameter	Value
Density of Tumor	1100 kg / m^3
Specific heat	4200 J /Kg .K
Absorption coefficient of tumor	cm^{-1} 0.115
Intensity of laser	0 - 0.2 w.cm^{-2}
Absorption efficiency	50.326
Time of exposure	140 sec
Maximum of Temperature	72.5 ºC

Table .5 Optical and biology properties of tissue at Kim et al study

when applying equations 24, 25 and 26 with the parameters used in the previous study, the results we obtained are illustrated in Figure 11 .The temperature reached in the presence of nanoparticles was 72.5 ºC ,and the left side of the figure 11 shows the temperature curve over time , with an exposure time of 140 second. Meanwhile , the left side of the figure represents the temperature curve in relation to tumor radius.

The results clearly demonstrate how the temperature increased significantly with the use of nanoparticles ,while before their application ,the temperature was close to 40 ºC .

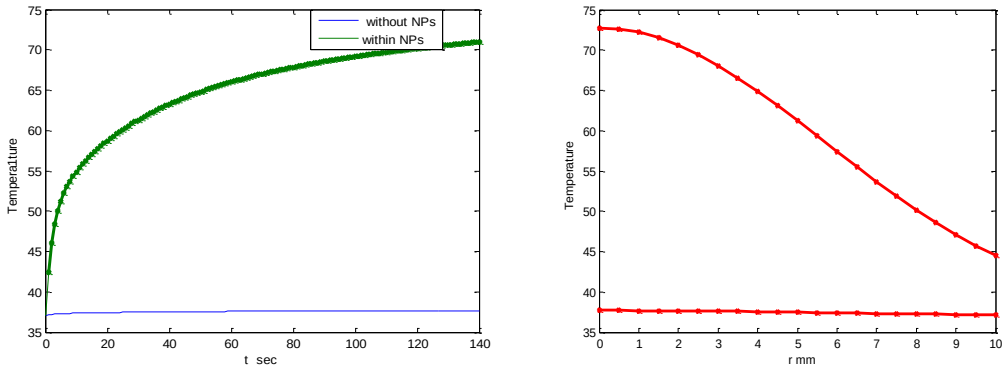


Fig .11 Temperature distribution for radius , and during 5 minutes at present study .

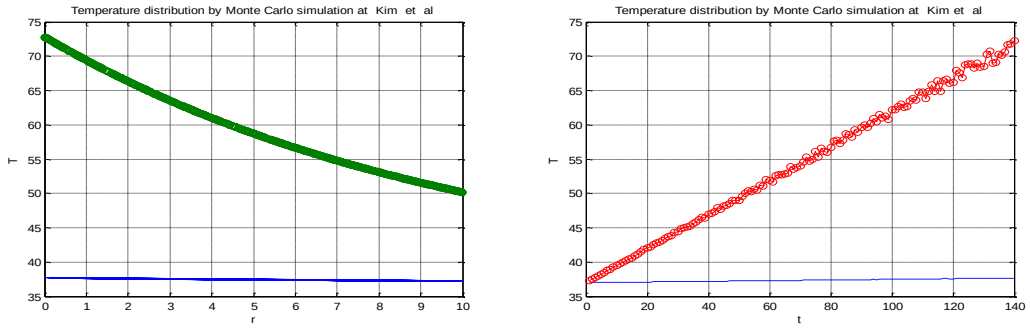


Fig . 12 Temperature distribution by Monte Carlo Simulation at present study.

When using Monte Carlo simulation ,we obtain the results shown in Figure 12 ,where the right side illustrates the temperature curve with respect to radius . the maximum temperature reached at the center of tumor is 72.5 ºC.The left side shows the temperature distribution over exposure time to radiation, with a temperature of 72.5 ºC at an exposure time of 140 seconds.

Based on the above, the curves in both figures 11 and 12 , which represent the temperature distribution within the tumor on the vertical axis, and the tumor radius and irradiation time on the horizontal axis, closely match the curves presented in the study by Kim et al. The content of the precious study can be referred to for a detailed comparison with the curves obtained in our work.

In the study conducted by Mooney et at (2017) .on a group of mice , the mice were divided into two groups: the first group was exposed to a laser intensity 1.4 w.cm^2 while the second group was exposed to a laser intensity of 2 w. cm^2 for a duration of 5 minutes, using the parameters outlined in table 6 .Rachael injected the group with a dose of 0 to 50 µ g of gold nanoparticles. The results obtained are illustrated in Fig 13[23]

Parameters	Values
ρ	1050 kg m^{-3}
C	3700 J $kg^{-1}K^{-1}$
μ_a	5.7 cm^{-1}
D	10 nm
ω_b	0.02 s^{-1}
λ	800 nm
I_{01}	1.4 w. cm^{-2}
I_{02}	2 w. cm^{-2}
t	5 min

Table - 6 optical and biologic Parameters at Mooney et al[23] .

In Rachael study, gold nanoparticles with dimension of [40 10] nm were used at concentration ranging from 0 to 50 µ g . Previous studies indicate that the absorption efficiency values for gold nanoparticles of that thickness range from 1.4 to 1.6 [13].It can refer to the study of Mooney and observed the shape of the distribution of temperature for tissue .

When applying the previous equation 24 ,25 ,26 , we obtained the temperature distribution at different concentrations . Figure 14 shows how the temperature is distributed with respect to the tumor radius, as will as Figure 15 shows the temperature distribution over an exposure time of 5 minutes with present study.

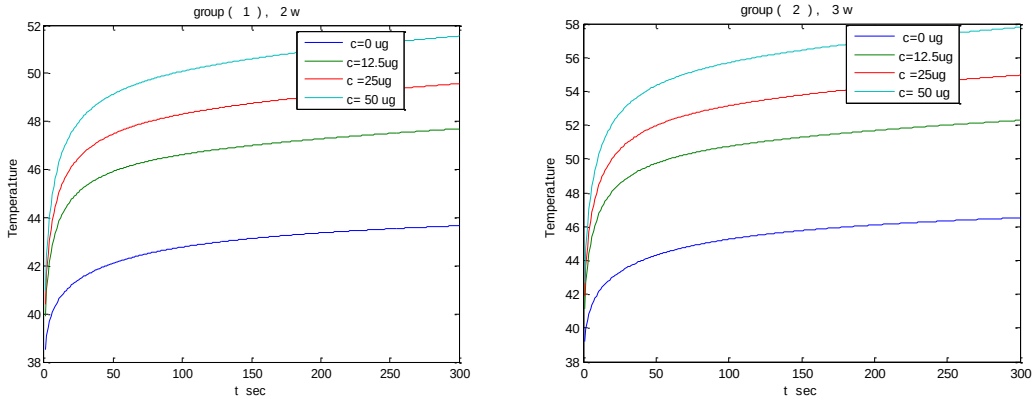
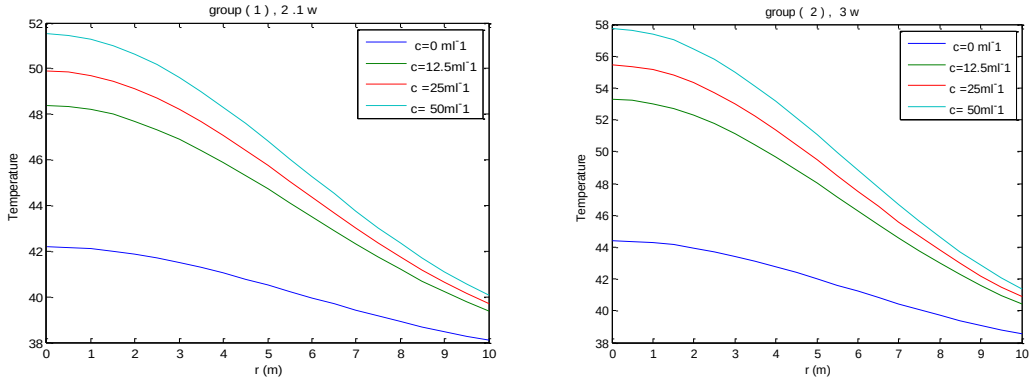


Fig .14 Temperature distribution during 5 minutes at present study



.

Fig. 15-a Temperature distribution in tumor during 5 minutes at present study .

By applying equation 24-b ,we obtain the change in tumor temperature with varying concentrations of the NPs . Figure 15-b shows the temperatures on the vertical axis corresponding to each NPs concentration at 2 and 3 watts.

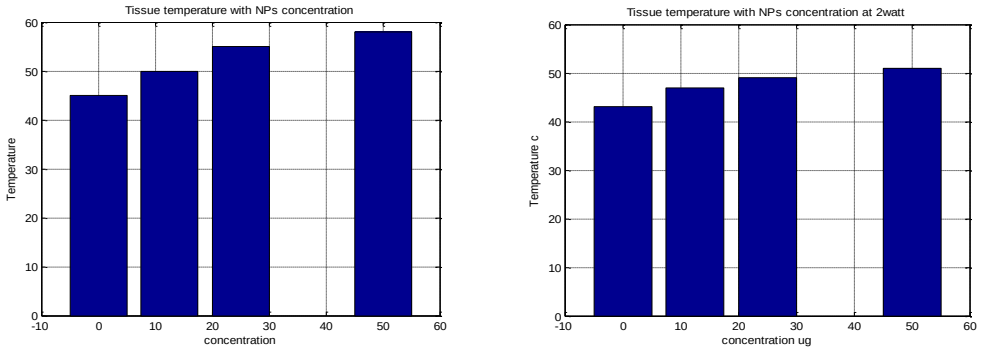


Fig .15-b Tissue temperature with NPs concentration at 2 watt and 3 watt .

The curves shown in figures 14 and 15-a , which illustrate the temperature distribution within the tumor as a function of irradiation time and tumor radius, were found to be in complete agreement with the curves presented in the study by Mooney et al.

In the study conducted by Karim et al (2020) .Which focused on the numerical Analysis of breast cancer and its localized mortality through gold nanoparticles and laser hyperthermia , rod -shaped gold nanoparticles with dimensions of [13 49] nm and cross-section absorption $5674nm^2$ were utilized . The temperature without nanoparticles reached 316 K ,which increased by 6 degrees when the laser was

applied .when gold nanoparticles were used , the temperature rose to 360 K .
When we applied the temperature distribution equation 25 and 26 ,the resalts obtained are illustrated in Figure 16 .The absorption efficiency of 13.5 was taken from multiple References , which provided the absorption efficiency values for Rod AuNPs and $\boldsymbol{g}$= 0.94 ,as shown in the following table.[24]

parameters	Value
Density of Tumor	1100 kg.m^{-3}
Specific heat	4200 J. $kg^{-1}K^{-1}$
Absorption coefficient of tumor	7 m^{-1}
Intensity of laser	100 w
Radius of laser	0.5 cm
Exposure of time	2 sec
Thermal conductivity	0.55 w / m .k
Metabolic heat	4200 wm^{-3}
Blood perfusion	6*$10^{-3}m^3$/kg.sec

Table -6 . parameters used in the study by Karim et al (2020)

Particle	λ (nm)	Size nm	Cross-section mm^2	Absorption efficiency	RF
Au NP	523	R= 20	2827	0.9 2.3	[13],[23] [16]
Rod AuNP	797	[13 49]	5674	13.5 13.8	[13],[23] [16]

Table -7 . parameters of the Rod Au NP used in the previous study and the current study.

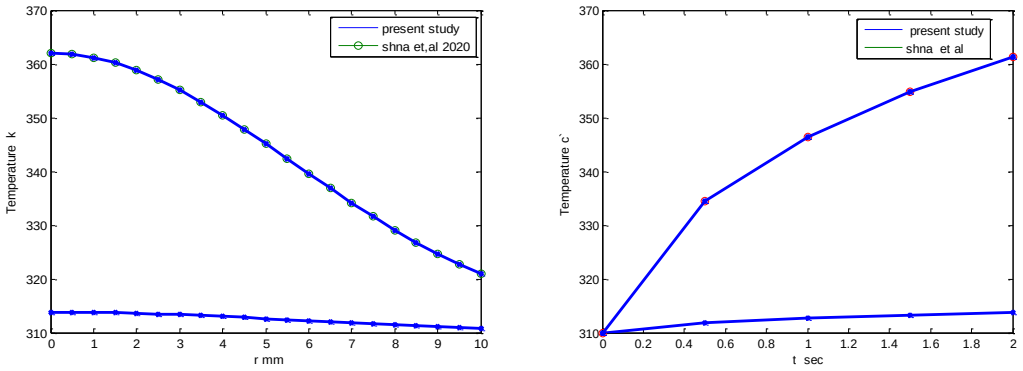


Fig .16 Temperature distribution within the tissue with and without nanoparticles
When applying mathematical equations 24 -26 ,we obtain the results shown in figure 16 , the results were consistent with the findings of the previous study.

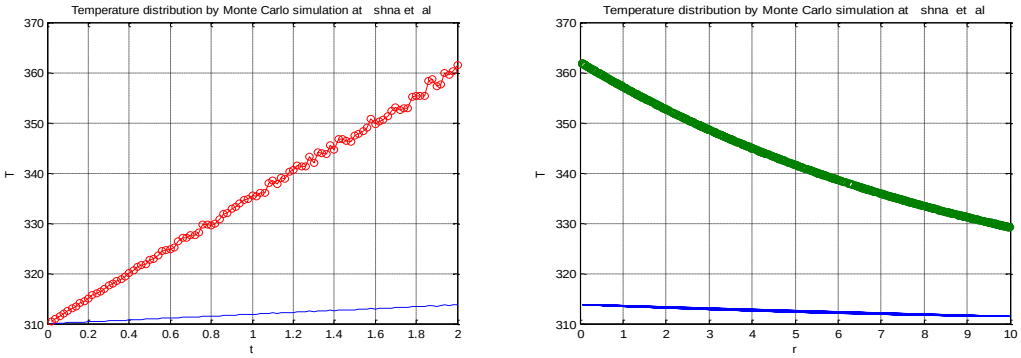


Fig. 17 Temperature distribution by Monte Carlo Simulation at present study .

When using Monte Carlo simulation ,we obtain the results shown in Figure 17 ,where the right side illustrates the temperature curve with respect to radius . the maximum temperature reached at the center of tumor is 361 Kelvin .The left side shows the temperature distribution over exposure time to radiation, with a temperature of 361 kelvin at an exposure time of 2 seconds.

Conclusion

Through the factors affecting the absorption efficiency of nanoparticles within the tissue ,such as particle concentration, size , tissue type, blood diffusion coefficient, tissue density ,and its thermal conductivity , it is essential to formulate a mathematical relationship that incorporates all these parameters .Even through we can use the factor in equation23,experimental and practical measurements must play a significant role in this relationship .Therefore , I hope researchers theoretical studies with experimental results ,providing more accurate insights into the distribution and thermal diffusion within tumors .while computational results are highly accurate ,they are simulations from which we must infer the physical parameters related to tumors to achieve a com-prehensive understanding that allows us to limit tumor spread within the body or predict future tumor occurrence .it is not as important to obtain a highly accurate temperature distribution curve as it is to effectively reduce the spread of a malignant tumor, even if it is just for one individual.

References

1- Yadav ,R ., Balaji .C ., Venkateshan .S.P .(2023).Radiative Heat Transfer in Participating Media with Matlab codes '. ,14,ANE Books Pvt .Ltd.,39

2- Haghighat Alireza .(2014) . Mote Carlo Methods for Particle Transport, ,CRC Press Tylor &Franceis Group, 125 ,129:130 , 143 ,188 ..

3- Mahan ,J.R .(2019)"The Monte Carlo Ray-Trace Method in Ragiation Heat Transfer and Applied Optics.Wiley &Sons Ltd,Hoboken ,NJ07030,USA,West Susser,PO198SQ,UK,32.

4- Asadi ,s. ,B,Leonardo., D,Martina., K,Sanzhar., S,Emiliano., S,Paola .(2020) .Laser-induced optothermal response of gold nanoparticles:Form a Physical viewpoint to cancer treatment application"Journal of Biophotonics ,DOI:10.1002/Jbio.202000161,16.

5- Pacella ,C.M ., Jiang ,T .,Mauri,G,(2020).Image-guided Laser Ablation.Gewerbestrasse 11, 6330 cham , Switzerland AG , 8-12, 146.

6- Mahan ,J.R.(2019) "The Monte Carlo ray-trace method in radiation heat transfer and applied optics: Hoboken , NJ: John Wiley &Sons ,30 .

7- Niemz ,H. Markolf .(2019)"Laser-Tissue Interactions Fundamentals and Applications,Fourth Edition,Switzerland AG ,

8- Wongchadakul,Patcharapon .,R, Phadungsak. (2020).Mathematical Modeling of Multilayered Skin with Embedded Tumor Through Combining Laser Ablation and Nanoparticles:Effects of Laser Beam Area, Wavelength , Intensity ,Tumor Absorption Coefficient and Its Position" International Journal of Heat and Technology Vol. 39 , No 1

9- Lapotko,D ., Lukianova,E .,Potapnev ,M. et al .(2006) .Method of laser activated nano-

thermolysis for elimination of tumor cells" Cancer Letters 239 ,37.
10- Ali , Moustafa R.k., Wu ,Yue ., El-Sayed ,Mostafa . A.(2019).Gold-Nanoparticle-Assisted Plasmonic Photothermal Therapy. J.Phys. C ,123,15378.
11- Hamid , Loutfy .,Madkour.(2022) '.Nanoparticle-based drug delivery in cancer treatment ,First edition .Boca Raton :CRC Press , 2-3.
12- Pustovalov,Victor .K .(2024).Heating of nanoparticles and their environment by laser radiation and applications"Medical Applications of Nanoscience and Nanotechnology,Prec.Eng 7 ,015001;doi:10.0022560, 3
13- Qin,Zhenpeng., Bishof,John .C .(2012).Thermophysical and biological responses of gold nanoparticle laser heating .Chem . Son .Rev ,41,1194-1196
14- K,Prashant ., S,L.Kyeong ., H,Ivan ., A.EL-Sayed,Mostafa .(2006).Calculated Absorption and Scatting Properties of Gold Nanoparticles of Different Size,Shape,and Composition :Applications in Biological Imaging and Biomedicine.J.Phys. Chem. B.Vol. 110 , No , 14
15- Boken,Jyoti., K,Parul ., T,Sheenam ., K,Dinesh. , P,Surendra'.(2017)'Plasmonic nanoparticles and their analytical applaction:A review" Applied Spectroscopy Reviews,VOL.52,NO.9,777.
16- Bayazitoglu,Yildiz. , Kheradmand ,Shiva., Tullius ,Toni .K.(2013) An overview of nanoparticle assisted laser therapy.nternational Journal of Heat and Mass Transfer 67 ,473
17- Dawei Wang , Wei Rod .(2022).Numerical Simulation on Thermal Response of Laser-Irradiated Biological Tissues Embsdded with Liquid Metal Nanoparticles. Journal of Thermal Science ,
18- Kapoor,Devesh .,M,Neha .,B,Anindita ., C,Mukesh .,Sharma ,P,Bhupendra ., M,Rahul.(2024).Polymer mediated light responsive therapeutics delivery system to treat cancer' . European Polymer Journal 210,112923 ,2.
19- Mohammedi ,A .,Bianchi,L .,Koraganbayev ,S., et al .(2022). Thermomechanical Modeling of Laser Ablation Therapy of Tumor :Sensitivity Analysis and Optimization of Influential Variables"IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING, VOL ,69,NO, 1 ,305.
20- Arfken.G.B., Webar , H.J.,& Harris,F.E.(2013). " Mathematical Methods for Physicists" Academic Press. 303-304
21- Xu,Xiao., M,Andrew., B,Yildiz .(2011) . Numerical investigation of nanoparticle-assisted laser-induced interstitial thermotherapy toward tumor and cancer treatments.Laser Med Sci 26
22- Kim,Moojoong ., K, Gwantaek ., K,Donghyuk., Y,Jaisuk .,K ,Dong-Kwon .(2019).Numerical Study on Effective Conditions for the Induction of Apoptotic Temperatures for Various Tumor Aspect Ratios Using a Single Continuous-Wave Laser in Photothermal Therapy Using Gold Nanorods . Cancers 11,764 ,7-8 .
23- Mooney ,Rachael ., S,Emiliano ., S,Paola ., Z,Ali .,A, Karen .,B, Jacob. M .(2016). Gold nanorod-mediated near-infrared laser ablation :in vivo experiments on mice and theoretical analysis at different settings. International Journal of Hyperthermia ,33:2, 152, 156.
24- Karim,Shna .A .,Hassan ,Yousif .M .(2020) .Numerical Analysiss of Breast Cancer Cell with Gold Nanoparticles Necrosis by Laser Hyperthermia . .Polytechnic Journal :Vol. 10:Iss. 2,Article 21, 140