Proper Conditions in Using of Eigenvectors Concepts in a Sensor Validation by Spectral Clustering Approach

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DOI: https://doi.org/10.56807/buj.v1i1.11

Abstract

We reported the proper use of eigenvector concept in spectral clustering approach such as the validation of sensors study[1]. The different cases of connection between sensors and references which lead to construct weighted matrix have been reported. As a result, a different eigenvector is produced. The proper conditions to get the correct distribution of sensor groups from eigenvectors value have been reported correctly in this paper. The study showed that the number of eigenvectors required to be used depends on some specific conditions of the number of sensors and targets along with their boundaries step functions .

الملخص

في هذا البحث وضحنا شروط الاستخدام المناسب لمفهوم المتجهات الذاتية في موضوع التجميع الطيفي [1] . الحالات المختلفة لطرق التوصيل بين الحساسات و المراجع الحساسة والتي بواسطتها نحصل على مصفوفة الثقل الأساسية درست في هذا البحث أيضا. كنتيجة لذلك تتولد متجهات ذاتية بقيم مختلفة. قدمنا في هذا البحث الشروط المناسبة للحصول على التوزيع الصحيح للمجموعات الحساسات وفقا للمتجهات الذاتية الناتجة. أوضحت الدراسة بأن العدد المطلوب من المتجهات الذاتية يعتمد على بعض الشروط المحددة من حيث عدد الحساسات والمراجع مع دوال الخطوة الحدودية لهم.

Keywords: *Spectral, Clustering, Matrix, Eigenvectors, Sensors*

Introduction

The solution of weighted matrix via eigenvalue and eigenvector is providing a full information about the goal problem. The application of these concepts (i.e. eigenvalue and eigenvector) is playing an important role in many fields such as physics, mathematics,…etc. As an example, in quantum mechanics the eigenvalues is used to specify the allowed energy levels while the eigenvectors represent the form of a function that describe the electron wave of each level. Another example is the determination of acoustic waves in cylindrical, spherical or rectangular cavities. The eigenvalues represent the exact values of allowed frequencies while the eigenvectors represent the wave equation/form mode of generated sounds in the cavities. Spectral clustering is a good technique to cluster a data based on some references. The technique has a wide application in many fields specially in manning data[1-7].

For sensors network that distributed in a sun explore place, the sensor performance decreases over time due to many reasons such as damaging by the sun temperature, breaking by the wind ….etc. Also, the rain can reduce or damage sensors. Even if the sensors get covered, with passage of time, the cover get lost. therefore, these sensors need to be checked periodically[1-2].

In this study, we reported different examples which contain different parameters. The main goal of these examples is to prove the correct steps in using of eigenvectors in spectral clustering approach as an effective tool to validation operation of sensors.

We started with an example reported by references 1&2 where 19 sensors and 3 reference targets were used with numbers 0-18 and 0-3, respectively. Furthermore, sensor i and target j were at locations 5i and 45j, respectively. We model sensor measurements of targets as a function of the location difference. i.e. the measurement of target j obtained at sensor i is a function of the difference between their location values as shown in Figure-1. However, we used same example factors (i.e. no. of sensors and no. of targets) varying the limits of step function. We solved the problem by three-step- function at five different boundaries.

Results and Discussion

This part is divided into four sections: The first deals with explanations of the construction of problem. The second deals with construction of step function and their effect on a produced weighted matrix. The third deals with construction of a weighted matrix. The fourth discusses the produced results of eigenvalues and eigenvectors of different example based on three-step-functions.

First: problem construction

We used the same example reported by[1-3] where 19 sensors and 3 reference targets were used with numbers 0-18 and 0-3, respectively. Furthermore, sensor i and target j were at locations 5i and 45j, respectively. We modeled sensor measurements of targets as a function of the location difference. i.e. the measurement of target j obtained at sensor i is a function of the difference between their location values as shown in Fig-1

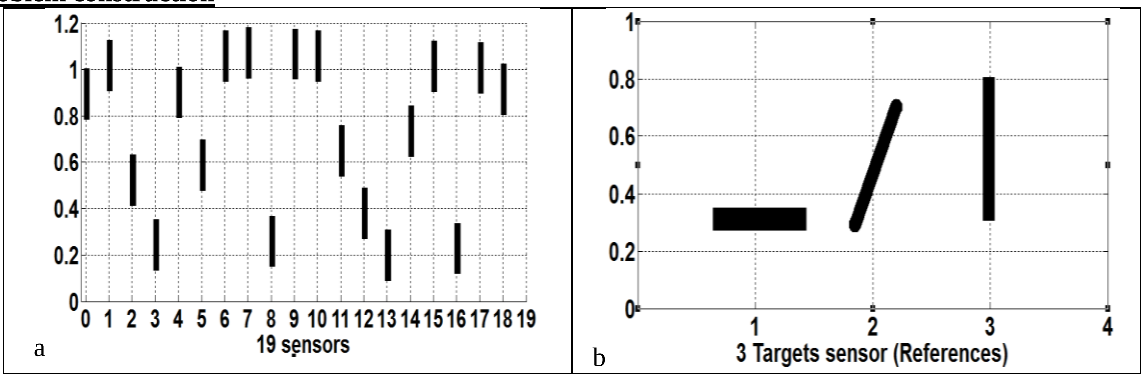


Fig-1: a) Sensors b) Targets

Second: Step function

Step function is the boundary conditions for each step that describe the range of strange relation between sensors and reference sensors. The step function limits are the bases for constructing the weighted matrix. Also, they are the bases in color representation of the relation between sensors and reference sensors.

Our study classified the limit of step function into three groups: **I, Interconnection way:** the way where there is some sensors connecting to more than one reference sensor. This method also is constructed in two ways. i.e. the regular method where same type of connecting lines of each reference sensor is connected to the same group of sensors(0-10,10-20,20-30), while the other one is a different type of connecting lines of each reference sensor to the same group of sensors(0-15,15-30,30-45). However, we studied both of cases. **II**, **Exact way:** where the last connected sensor to first reference is not connected to the second reference and vice versa. In other words, there is no interconnection between any reference sensors and at the same time there no sensor is connected to any reference sensor. Also we can say that each sensor has one line connection to a reference sensor (0-8,8-16,16-24). **III, Some Disconnecting way:** in this method, some sensors are not connected to any reference sensor. Here we used two examples: in the first the disconnected sensors are in the following order (0-6,6-12,12-18) while in second example they are not in the same order . i.e.(0-6,6-12,18-24).

Third, Construction of a Weighted Matrix

According to step function values, matrix A is constructed having a number of rows equal to the number of reference sensors while the number of column is equal to the number of sensors.

Now for constructing the weighted matrix, we multiplied the matrix A by its transport which produced a weighted matrix 19X19.

Fourth: Produced Eigenvalues and Eigenvectors

The produced eigenvalues are three values which are related to the three references. The eigenvectors are used to describe the connection methods and groups for sensors and reference sensors. These methods are used to identify the bad sensors in the whole example. In reference paper [1] the first and second eigenvectors were used. In this study it will be proved that the first and second eigenvectors are not sufficient for most cases to describe the solution of the problem. We started based on three-step- functions.

The First example: Interconnecting Case (regular[1]-irregular)

In this way، some sensors are connected to more than one reference sensors. The steps are as follows :

Step 1: We fixed the number of targets (Reference sensors) 0-3 where 0=0,1=45, 2=90

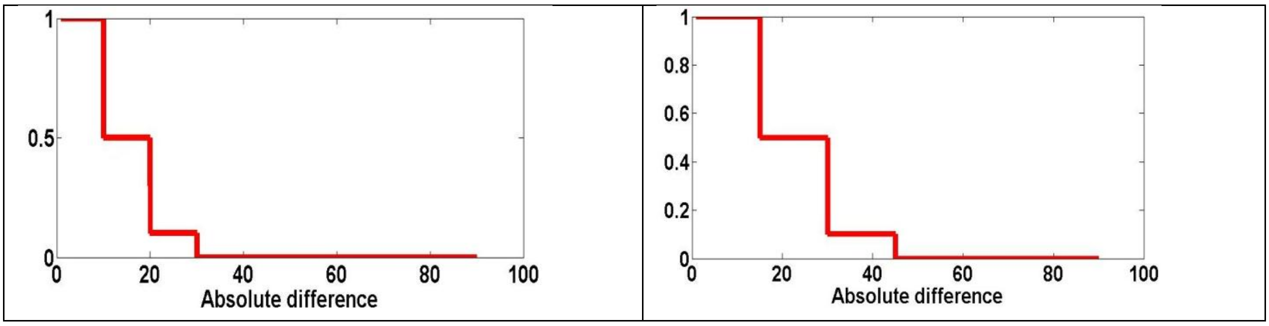
Step 2: we fixed the number of sensors 0-18 where each sensor has an angle value =5i

Step 3 (**Regular)** : we used values where each step represents the difference value for each sensor with each target, (1for x <10 , 0.5 for 10=<x<20, 0.1 for 20=<x<30 and 0 for x=>30)

(Irregular): (1for 0=<x<15, 0.5 for 15=<x<30, 0.1 for 30=<x<45 and 0 for x=>45)

The step function and weighted matrix are shown in fig-2

Regular	Irregular



Regular	$A = \begin{pmatrix} 1\ 1\ 0.5\ 0.5\ 0.1\ 0.1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0.1\ 0.1\ 0.5\ 0.5\ 1\ 1\ 1\ 0.5\ 0.5\ 0.1\ 0.1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.1\ 0.1\ 0.5\ 0.5\ 1\ 1 \end{pmatrix}$
Irregular	$A = \begin{pmatrix} 1 & 1 & 1 & 0.5 & 0.5 & 0.5 & 0.1 & 0.1 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.1 & 0.1 & 0.5 & 0.5 & 0.5 & 1 & 1 & 1 & 1 & 1 & 0.5 & 0.5 & 0.5 & 0.1 & 0.1 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.1 & 0.1 & 0.5 & 0.5 & 0.5 & 1 & 1 & 1 \end{pmatrix}$

Fig-2: Step Function and weighted matrix

The eigen values of the weighted matrix in case of regular are 4.0405-2.52-2.5195 while in case of irregular are 7.1530-3.78-3.187. The eigenvectors of the weighted matrix in case of regular and irregular are presented in fig-3

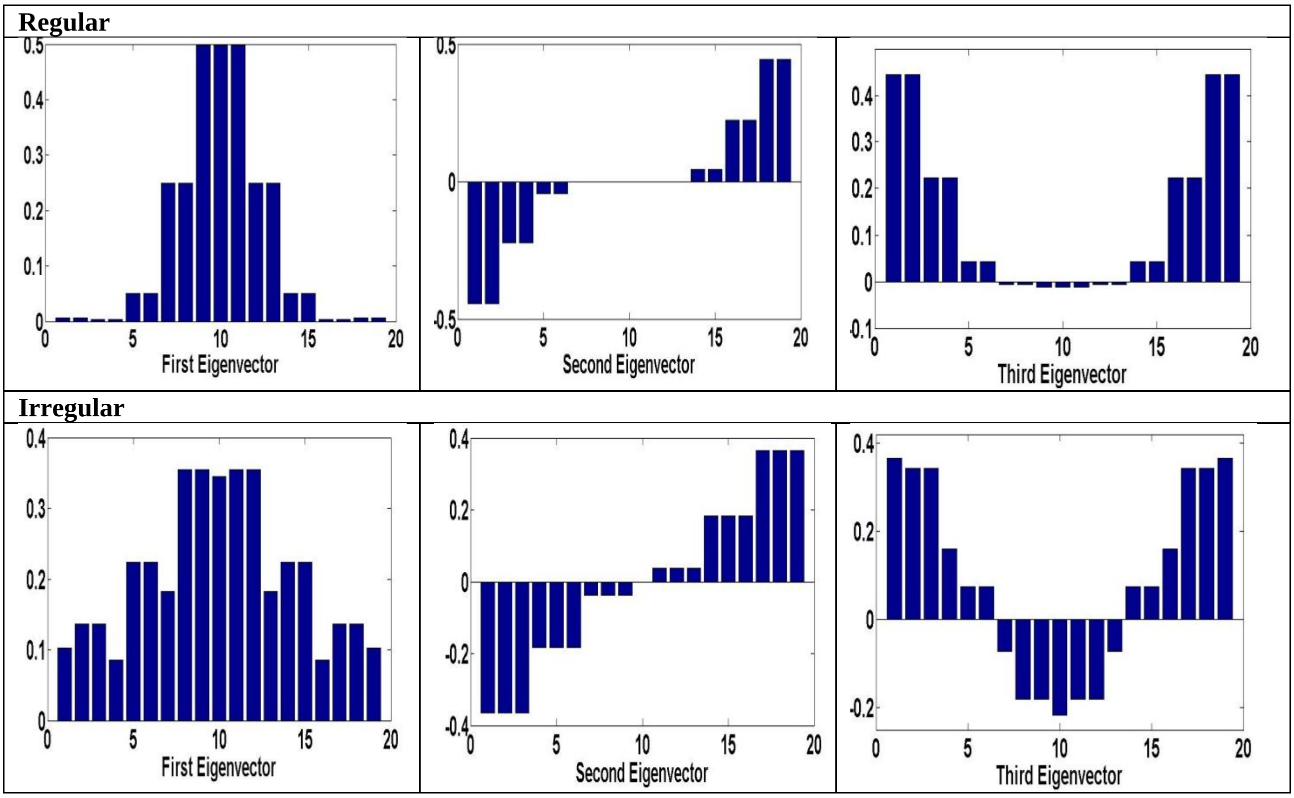


Fig-3 the three eigenvectors of regular and irregular examples

From the color system, it can be seen that the cluster groups [from reference [1]] and from eigenvectors have the same results. Here it can be observed that the first and second eigenvectors are sufficient to represent the clustering groups of sensors. However, it can be seen that this statement cannot be generalized for the other cases.

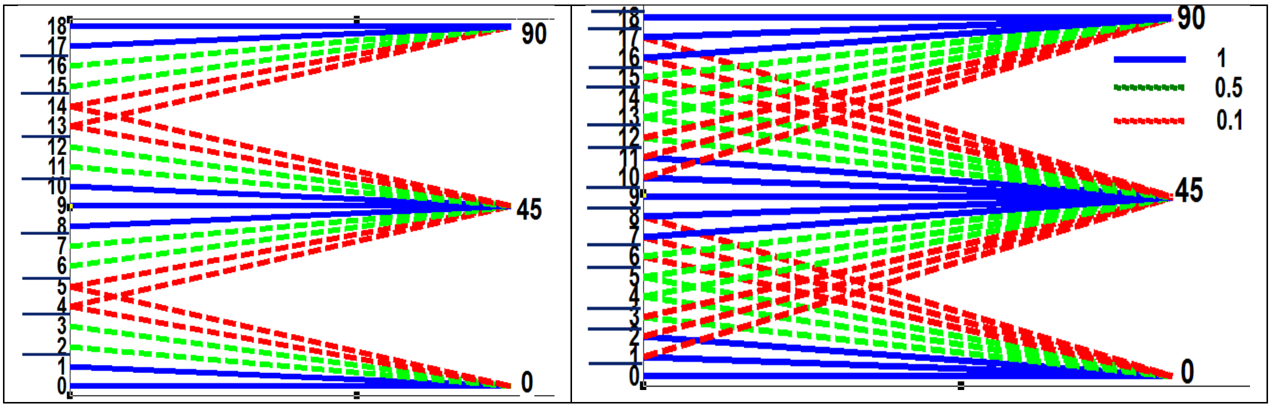


Fig-4: color representation

In the irregular case, from the color system (see fig-4), the thirteen clustering groups are: [0][12][3][45][6][78][9][1011][12][1314][15][1617][18]

Now let us study the clustering groups from the three eigenvectors. The groups from first eigenvectors are: [0][12][3][45][6][78][9][1011][12][1314][15][1617][18] with identical values of the first half of the groups (up to 9 sensors) to the second half. However, this identical value refers to the identical strength of the relation of the first half of the groups (up to 9 sensors) with the second half such as group [0] and [18] which have the same strength relation and so on.

The second eigenvector gives the following seven cluster groups: [012][345][678][9][101112][131415][161718]. The difference between these clustering groups and the groups from the color system and the first eigenvectors is very clear. So, it is clear that the violet of first example statement that mentions the sufficient value of the first and second eigenvectors to identify the cluster groups of the problem. lt can be noted that the second eigenvector divides the clustering groups into two big groups with negative and positive values around the medium value, that is, group[9]. At the same time, it ignores the connection of medium reference sensors(here 45) with sensors and takes only the connection relation of the other reference sensors around the medium one (here references 0 and 90) . So, it can be seen that the sensors 0,1,2 have the same connection relation strength with references 0 and so on.

The third eigenvectors provides the following thirteen clustering groups: [0][12][3][45][6][78][9][1011][12][1314][15][1617][18]. It is clear that, the third eigenvector provides similar groups of the first eigenvector dividing them into three big groups; the first and third are with positive values while the second group [6][78][9][1011][12] is with negative values.

Therefore in case of interconnection with irregular relation, the information from eigenvectors are not identical and provide different clustering groups according to the corresponding reference sensors.

Signal system

1- Interconnecting –regular

Table-1: Values of last three columns of eigenvectors in case of interconnecting-regular

0.4	0.4	0.2	0.2	0.04	0.04	-0	-0	-0	-0	-0	-0	-0	0.04	0.04	0.2	0.2	0.4	0.4
0.4	0.4	0.2	0.2	0.04	0.04	-0	-0	-0	-0	-0	-0	-0	-0.04	-0.04	-0.2	-0.2	-0.4	-0.4
0.006	0.006	0.003	0.003	0.05	0.05	0.2	0.2	0.4	0.4	0.4	0.2	0.2	0.05	0.05	0.003	0.003	0.006	0.002

We can observe that the table-1 shows the three groups: (+ ++=2, - - +=1, + - +=0) (Interconnection-regular), so we can represent Table-1 :

2	**2**	**2**	**2**	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**

2- Interconnecting –irregular

Table-2: Values of last three columns of eigenvectors in case of interconnecting-irregular

0.3	0.3	0.3	0.1	0.07	0.07	-0.07	-0.1	-0.1	-0.2	-0.1	-0.1	-0.07	0.07	0.07	0.1	0.3	0.3	0.3
0.3	0.3	0.3	0.1	0.1	0.1	0.03	0.03	0.03	-0	-0.03	-0.03	-0.03	-0.1	-0.1	-0.1	-0.3	-0.3	-0.3
0.1	0.1	0.1	0.08	0.2	0.2	0.1	0.3	0.3	0.3	0.3	0.3	0.1	0.2	0.2	0.08	0.1	0.1	0.1

3	**3**	**3**	**3**	**3**	**3**	**2**	**2**	**2**	**1**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**

Thre

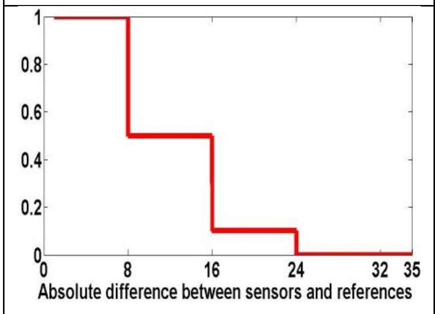
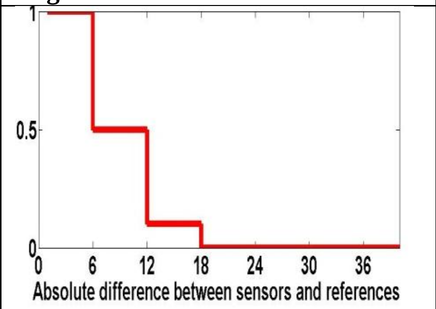
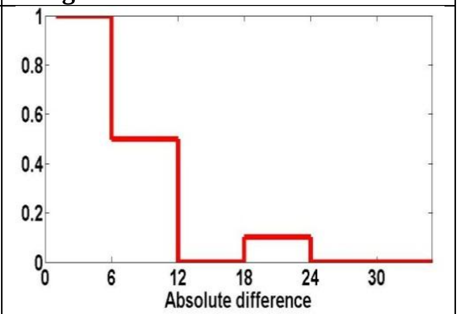
e groups: (+ ++=3, - + +=2,- - +=1 + - +=0) (Interconnection-irregular)

From the both tables of a signal system it can be observed that the number of signal groups are three in both cases which means that the regular and irregular connection has no effect on a signal method.

The second example: Exact and Disconnect Case (regular-irregular)

In the exact case, each sensor has one line connection to a reference sensor which means that there is no interconnection between reference sensors. Here we used a step function limit as: 1for 0=<x<8, 0.5 for 8=<x<16, 0.1 for 16=<x<24 and 0 for x=>24

In the disconnection case, there are some sensors which are not connected to any reference sensor. The case that the disconnected sensors are infollowing orders called **Regular disconnection** and we used a step function limit as: 1for 0=<x<6, 0.5 for 6=<x<12, 0.1 for 12=<x<18 and 0 for x=>18 while in case of no following orders it call **Irregular disconnection** and we used a step function limit as: 1for 0=<x<6, 0.5 for 6=<x<12, 0.1 for 18=<x<24 and 0 elsewhere.

Exact case	**Disconnection case**	
	Regular	**Irregular**
		

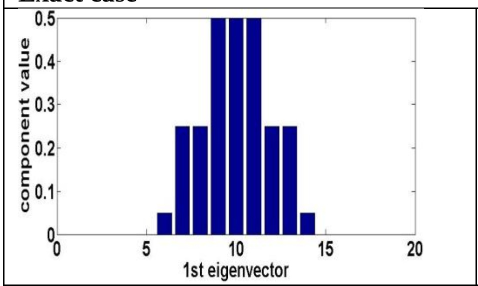
Exact	$A = \begin{pmatrix} 1\ 1\ 0.5\ 0.5\ 0.1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0.1\ 0.5\ 0.5\ 1\ 1\ 1\ 0.5\ 0.5\ 0.1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.1\ 0.5\ 0.5\ 1\ 1 \end{pmatrix}$
Disc. regular	$A = \begin{pmatrix} 1\ 1\ 0.5\ 0.1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 0.1\ 0.5\ 1\ 1\ 1\ 0.5\ 0.1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.1\ 0.5\ 1\ 1 \end{pmatrix}$
Disc. irregular	$A = \begin{pmatrix} 1\ 1\ 0.5\ 0\ 0.1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0.1\ 0.5\ 0\ 1\ 1\ 1\ 0.5\ 0\ 0.1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0.1\ 0\ 0.5\ 1\ 1 \end{pmatrix}$

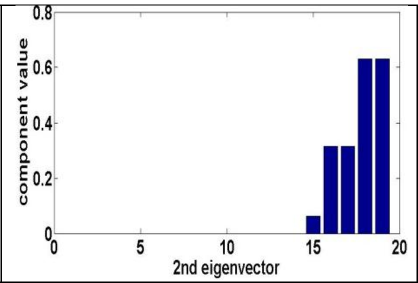
Fig-5: Step Function and weighted matrix

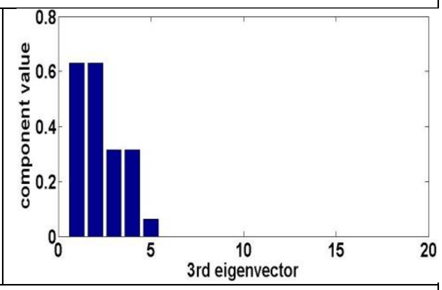
The eigenvalues of a weighted matrix in exact case are 4.02-2.51-2.51

The eigenvectors of a weighted matrix in case are presented in fig-6

Exact case







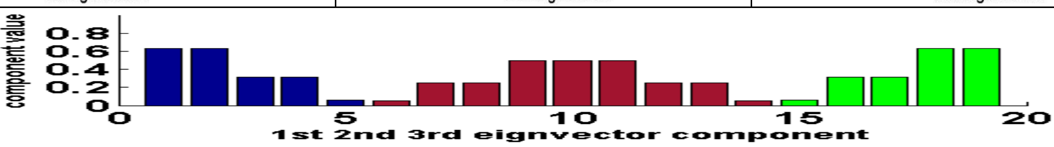


Fig-6 the three eigenvectors of regular and irregular examples

The eigenvectors of the weighted matrix in case of disconnected (regular) are: 3.52-2.26-2.26 and irregular are 3.52-2.26-2.26. The eigenvectors are presented in fig-000000 The color system for exact case provides the following groups: [01][23][4][5][67][8910][1112][13][14][1516][1718]. Note that, there is no intercrossing of connection between reference sensors.

From the eigenvectors of exact case we found that the cluster groups from the first eigenvector: [5][67][8910][1112][13]. It represents the connections for the second reference sensor (i.e,45) only. The second eigenvector provides [01][23][4] and the third eigenvector provides[14][1516][1718]. They represent the connection from the first and third reference sensors separately.

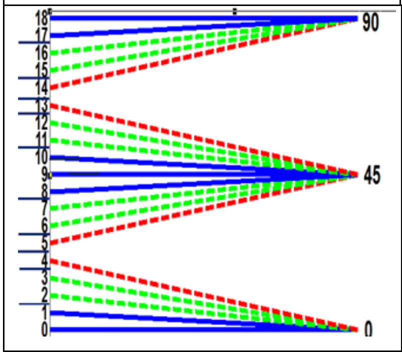
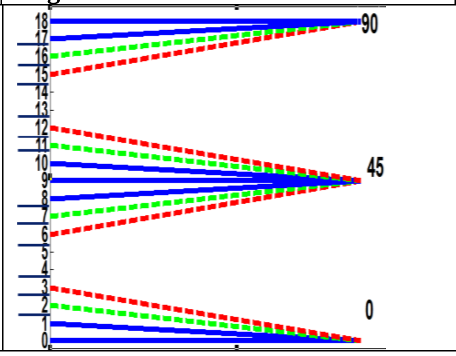
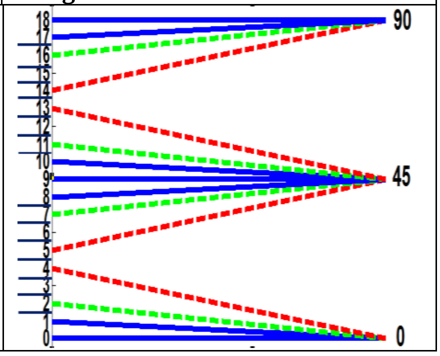
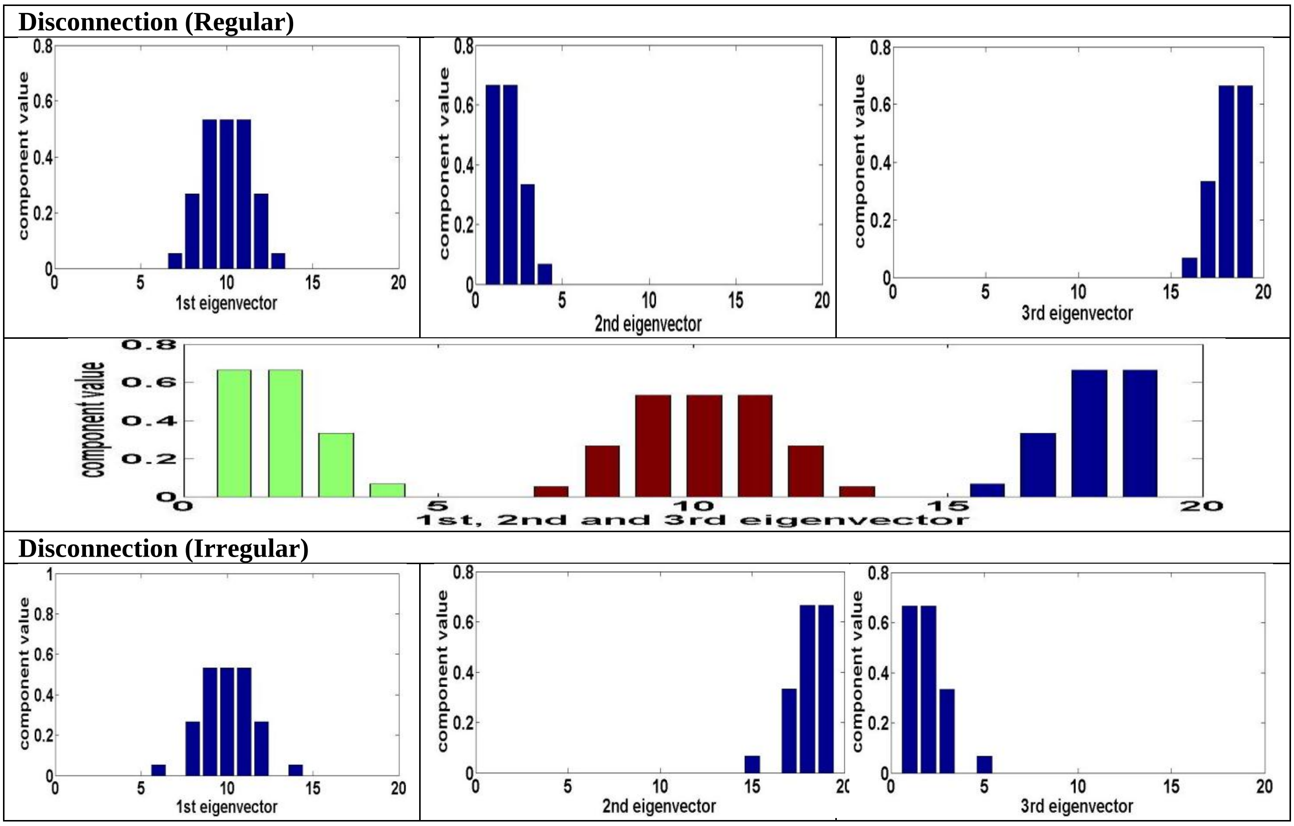
Exact case	Disconnection case	
	Regular	Irregular
		

Fig-7: color representation system

Here, we cannot extracted the full clustering groups from one eigenvector either the first, second or third one. In order to obtain the full cluster groups from eigenvector values, we need to use all eigenvectors in one graph or gather all cluster groups from the three eigenvectors as shows in fig-8



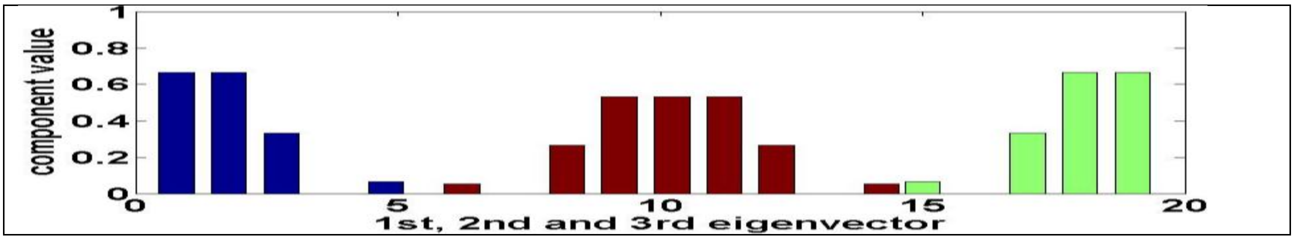


Fig-8: The three eigenvectors of regular and irregular examples in disconnecting case

The color system for disconnection-regular case provides the following thirteen groups: [01][2][3][45][6][7][8910][11][12][1314][15][16][1718]. Note that, there are two groups [45] and [1314] which are not connected to any reference sensors.

From the eigenvectors of disconnection-regular case we found that the cluster groups from the first eigenvector: [6][7][8910][11][12]. It represents the connections for the second reference sensor (i.e,45) only. The second eigenvector provides [01][2][3] and the third eigenvector provides [15][16][1718]. They represent the connection from the first and third reference sensors separately. The disconnecting groups disappear from all the three eigenvectors values.

However, a similar case to the exact case is that we cannot extract the full clustering groups from one eigenvector either the firs, second or third one but need to gather all eigenvectors in one graph or to gather all cluster groups from the three eigenvectors as shown in fig-0000

The color system for disconnection-irregular case provides the following fifteen groups: [01][2][3][4][5][6][7][8910][11][12][13][14][15][16][1718]. Note that, there are four groups [3][6] [12]and[15] which are not connected to any reference sensors.

From the eigenvectors of disconnection-irregular case, we found that the cluster groups from the first eigenvector are: [5][7][8910][11][13]. They represent the connections for the second reference sensor (i.e,45) only. The second eigenvector provides [01][2][4] and the third eigenvector provided [14][16][1718]. They represent the connection from the first and third reference sensors separately. The disconnecting groups disappear from all the three eigenvectors values.

Here also we found a similar case to the exact and disconnection-irregular case that we cannot extract the full clustering groups from one eigenvector either first, second or third one but need to gather all eigenvectors in one graph ore to gather all cluster groups from the three eigenvectors as shows in fig-0000

Signal system

1- Exact

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.063	0.31	0.31	0.63	0.63
0.63	0.63	0.31	0.31	0.063	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0.049	0.24	0.24	0.49	0.49	0.49	0.24	0.24	0.049	0	0	0	0	0

Three groups: (0 + 0=2, 0 0 +=1, + 0 0=0) (Exact)

From the exact case table of a signal system, it can be observed that the number of signal groups are three which means that the exact connection has no effect on a signal method.

2-Disconnection (regular)

0.6	0.6	0.3	0.06	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.06	0.3	0.6	0.6
0	0	0	0	0	0	0.05	0.2	0.5	0.5	0.5	0.2	0.05	0	0	0	0	0	0

Three groups + two for disconnecting sensors: (+ 0 0=4, 0 0 0=3, 0 0 +=2, 0 0 0=1, 0 + 0=0) (disconnecting-regular)

3-Disconnection (irregular)

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.06	0	0.3	0.6	0.6
0.6	0.6	0.3	0	0.06	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0.05	-0	0.2	0.5	0.5	0.5	0.2	0	0.05	0	0	0	0	0

Three groups + two for disconnecting sensors: (0+ 0=4, 0 0 0=3, 4, 0 0 +=2, 0 0 -=1 ,2, 3, 2, + 0 0=0) (disconnecting-regular)

From the both tables of a signal system it can be observed that the number of signal groups are different and more than three in both cases which means that the regular and irregular connection has a big effect on a signal method.

Conclusion

We successfully studied the proper conditions for using eigen values and eigenvectors in spectral clustering. We have applied the method on five examples which make all the possibilities of using eigenvectors clear. Also, we found that similar results for the exact and disconnection cases. That is to say, we cannot extract the full clustering groups from one eigenvector either first, second or third one but need to gather all eigenvectors in one graph or to gather all cluster groups from the three eigenvectors. However, in interconnection cases the one eigenvector can provide full information about cluster groups according to the number of references. We demonstrate also the effect on a signal system in disconnection case only.

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